1993 ALGEBRA PRELIMINARY EXAMINATION

This exam is divided into three sections: Group Theory, Ring Theory and Galois Theory. In each section you are required to do two problems.

Ring Theory

1. Show that $J(Mat_n(R)) = Mat_n(J(R))$ for any ring R.

2. State the Artin-Wedderburn Theorem and outline its proof.

3. Let R be a domain. Recall that a nonzero element $a \in R$ is *irreducible* if it is not a unit, and whenever $a = bc$ for $b, c \in R$, then either b is a unit or c is a unit. If R is Noetherian, show that every nonzero nonunit element of R is a product of irreducible elements.

Group Theory

1. Show that every group of order 56 is not simple.

2. For a subgroup U of a group G, define $N_G(U) = \{g \in G : g^{-1}Ug = U\}$. If P is a p-Sylow subgroup of a finite group G, show that $N_G(N_G(P)) = N_G(P)$.

3. Find (up to isomorphism) all abvelian groups of order 3528.

Galois Theory

1. Show that a finite subgroup of the multiplicative group of a field is cyclic.

2. Determine the Galois group of the polynomial $x^3 - 11 \in \mathbf{Q}[x]$.

3. Prove or give a counterexample: Every field extension is separable.

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