Circa 1990 General Examination in Analysis (administered by J. B. Brown).

Answer at least 8 questions:

CONTINUITY AND DIFFERENTIABILITY

1. Describe a continuous nowhere differentiable $f\colon[0,1] \to R$.
2. Describe a differentiable $f\colon[0,1] \to R$ such that f' is not continuous. Can f' be totally discontinuous?
3. For each n, describe
   (1) an n-times differentiable $f\colon[0,1] \to R$ such that $f^{(n)}$ is not continuous, and
   (2) an n-times continuously differentiable $f\colon[0,1] \to R$ which is not $(n+1)$-times differentiable.

MEASURE AND CATEGORY

4. Define what it means to say that a subset of R is of Lebesgue measure zero and what it means to say it is of first category.
5. Prove that a countable subset of R is of Lebesgue measure zero and of first category.
6. Give an example of first category subset of [0,1] which is of Lebesgue measure 1.

MEASURABLE FUNCTIONS

7. If A is a sigma algebra of subsets of some set Ω, define what it means to say that a function $f\colon\Omega \to R$ is measurable with respect to A.
8. Prove that if $f_1, f_2, \ldots$ is a sequence of A-measurable functions converging pointwise to a function f, then f is A-measurable.

RIEMANN AND LEBESGUE INTEGRALS

9. Define the Riemann and Lebesgue integrals of a function $f\colon[0,1] \to R$.
10. Give an example of a bounded Baire-1 function which is not Riemann integrable. Explain why such a funtion would have to be Lebesgue integrable.
11. Give an example of a derivative on [0,1] which is not Lebesgue integrable.

L^p SPACES

12. Define $L^p[0,1]$ and $L^p(R)$ for $p > 0$.

13, Prove that $L^2[0,1] \subseteq L^1[0,1]$.

14, Show that $L^2(R) \nsubseteq L^1(R)$ and $L^1(R) \nsubseteq L^2(R)$.