STAT 7600/7610 Mathematics Statistics Preliminary Exam, August 16, 2013

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Name:______________________________

1. It is a closed-book in-class exam.

2. Calculator is allowed.

3. Show your work to receive full credits. *Highlight your final answer.*

4. Solve any **five** problems out of **seven**.

5. Total points are **50** with **10** points for each problem.

6. Time: 180 minutes. (8:00am - 11:00am, August 16th, 2013)

1	2	3	4	5	6	7	Total

1. Suppose that $X_1, ..., X_n$ be iid random variables of Poisson(λ).

(a) Find the best unbiased estimator of $P(X = t) = \lambda^t e^{-\lambda}/t!$, $t = 0, 1, 2, \ldots$.

(b) For the best unbiased estimators of $P(X = 1) = \lambda e^{-\lambda}$, calculate the asymptotic relative efficiency with respect to maximum likelihood estimate.

2. Let $X_1, ..., X_n$ be an iid sample from $N(\mu, 1)$ and $Y_1, ..., Y_m$ be an iid sample from $N(\theta, 1)$. Assume $X_1, ..., X_n$ and $Y_1, ..., Y_m$ are independent. Consider hypothesis

$$H_0 : \mu - \theta = 1, \quad v.s. \quad H_1 : \mu - \theta \neq 1.$$

(a) Find the likelihood ratio test (identify test statistic and rejection region).

(b) Show that the rejection region in (a) can be represented in terms of $|\bar{x} - \bar{y} - 1|$.

3. Let $X_1, ..., X_n$ be iid random variables of Poisson($\sqrt{\lambda}$) where $\lambda > 0$.

(a) Construct the uniformly most powerful (UMP) level α test of $H_0 : \lambda_0 = 1$ vs $H_1 : \lambda_0 > 1$. If $n = 1$ and $\alpha = 0.05$, what is the reject region of the test?

(b) If $\hat{\lambda}_n$ is the maximum likelihood estimate for λ, find the limiting distribution of $\sqrt{n}(\hat{\lambda}_n - \lambda)$.

4. Suppose $X_1, \ldots, X_n$ are independent random variables with $P(X_i = 1) = p = 1 - P(X_i = 0)$. Consider the model with $0 < p < 1$.

(a) Find a complete and sufficient statistic T.

(b) Find the UMVUE of $var(X_1)$.

5. X and Y are independent random variables with $X \sim$ exponential (λ) and $Y \sim$ exponential (μ). Define $Z = \min\{X, Y\}$ and $W = 0$, if $Z = X$ and $W = 1$, if $Z = Y$.

(a) Find the joint distribution of Z and W.

(b) Prove that Z and W are independent.

6. Suppose that $X_1, \ldots, X_n$ are iid Geometric(θ) random variables with probability mass function

$$P(X_i = x) = \theta(1-\theta)^x, \quad x = 0, 1, 2, \ldots.$$

(a) Let $T_n = X_1 + \cdots + X_n$. Find the probability mass function of T_n.

(b) Show that

$$U = \begin{cases} 1, & X_1 \leq x \\ 0, & X_1 > x \end{cases}$$

is an unbiased estimate of the cumulative distribution of X_1.

(c) Find the UMVUE of the cumulative distribution of X_1 based on data $X_1, \ldots, X_n$.

7. Suppose that $X_1, \ldots, X_n$ are independent and identically distributed with cumulative distribution function

$$F(x; \theta, \phi) = \begin{cases} 0, & x < \phi \\ 1 - (x/\phi)^{-\theta}, & x \geq \phi \end{cases}$$

(a) Find the distribution of $Y_i = \log(X_i/\phi)$.

(b) Find the maximum likelihood estimate of (θ, ϕ).