STAT 7600/7610 Mathematics Statistics Preliminary Exam, August 15, 2014

Statistics Group, Department of Mathematics and Statistics, Auburn University

Name:________________________________

1. It is a closed-book in-class exam.

2. Calculator is allowed.

3. Show your work to receive full credits. *Highlight your final answer.*

4. Solve any **five** problems out of **seven**.

5. Total points are **50** with **10** points for each problem.

6. Time: 180 minutes. (8:00am - 11:00am, August 15th, 2014)

1	2	3	4	5	6	7	Total

1. Find the $1-\alpha$ confidence set for a that is obtained by inverting the LRT of $H_0 : a = a_0$ vs $H_0 : a \neq a_0$ based on a sample $X_1, ..., X_n$ from a $N(\theta, a\theta)$ family, where θ is unknown.

2. Let $X_1, ..., X_n$ be an iid sample from $N(\mu_x, \sigma_x^2)$ and $Y_1, ..., Y_m$ be an iid sample from $N(\mu_y, \sigma_y^2)$. Assume $X_1, ..., X_n$ and $Y_1, ..., Y_m$ are independent. Assume all parameters $\mu_x, \mu_y \in R$, $\sigma_x, \sigma_y \in (0, +\infty)$ are unknown and $n > 1$ and $m > 1$.

(a) Find the UMVUE of $\mu_x - \mu_y$.

(b) If $\sigma_x = \sigma_y = \sigma$, find the UMVUE of $\mu_x - \mu_y$ and σ, and determine if the UMVUE of σ achieves the CR lower bound.

3. Let X be one observation from a pdf f on $(0, +\infty)$, $0 < \alpha < 1$ and let $f_1(x) = e^{-x}$, $f_2(x) = xe^{-x}$, $x > 0$. To test $H_0 : f = f_1$ vs $H_0 : f = f_2$ based on X, find an uniformly most powerful (UMP) test statistic T and compute its power.

4. Let $X_1, ..., X_n$ be an iid sample from $N(\mu, \sigma^2)$ distribution where $\mu \in R$, $\sigma \in (0, +\infty)$. Consider the estimation of σ^2 with the squared error loss. Show that $\frac{n-1}{n}S^2$ is better than S^2, the sample variance. Find an estimator of the form cS^2 with a constant c such that it is better than $\frac{n-1}{n}S^2$?

5. Let $X_1, ..., X_n$ be iid random variables having the uniform distribution on the interval (a, b), where $-\infty < a < b < \infty$. Show that $\left(X_{(i)} - X_{(1)}\right) / \left(X_{(n)} - X_{(1)}\right)$, $i = 2, \ldots, n-1$, are independent of $\left(X_{(1)}, X_{(n)}\right)$ for any a and b.

6. Suppose that $X_1, \ldots, X_n$ are iid samples from $N(0, \sigma^2)$, with unknown parameter $\sigma \in (0, +\infty)$. Find the asymptotic relative efficiency of $\sqrt{\pi/2}\sum_{i=1}^n |X_i| / n$ with respect to $\left(\sum_{i=1}^n X_i^2/n\right)^{1/2}$.

7. Let $X_1, ..., X_n$ be an iid sample from $N(\mu, \sigma^2)$ with an unknown μ and known σ^2. Suppose that the prior distribution on μ is $N(\theta, \tau^2)$ and θ and τ are known. Find the posterior distribution of μ, $E(\mu|X_1, \ldots, X_n)$ and $Var(\mu|X_1, \ldots, X_n)$.