TOPOLOGY PRELIMINARY EXAMINATION
1991

Except as noted with (*) you may state without proof material from Set Theory and Algebra.

I.

1. Define
   (a) Hausdorff space
   (b) compact
   (c) connected
   (d) locally connected
2. Show that a compact subset of a Hausdorff space is closed.
3. Give an example of a connected space which is not locally connected.

II.

1. Define the set-theoretic product of a family of sets.*
2. Define the product topology on a family of spaces.
3. Given a countable family of metric spaces, give a metric for the product topology of these spaces.

III

1. Define
   (a) locally Euclidean space
   (b) (topological) manifold
   (c) partition of unity
2. Show that in a manifold every open cover has a partition of unity subordinated to it with the further property that each function has a compact support.

IV

1. Define a covering map (Spanier: covering projection)
2. Prove the Uniqueness of Lifting Theorem for maps of a connected space into the base space of a covering map.

V

1. State the Lifting Map Theorem for covering space relating the lifting problem for a map with its effect on fundamental groups.
2. Without giving a proof, state how to construct a lift.

VI

1. Show that a proper map of a connected space onto a Euclidean space which is a local homeomorphism is a homeomorphism.
2. State a generalization (of the above) for "locally path-connected, simply connected space" replacing "Euclidean space."

VII

1. Define cofibration.
2. Show that if the inclusion of A into X is a cofibration and A is contractible, then $X \to X/A$ is a homotopy equivalence.

VIII

1. Give a sketch of proof that a map $f : X \to Y$ is a homotopy equivalence iff X is a strong deformation retract of the mapping cylinder of f.
2. What is the homotopy type of the one-point compactification of the product of locally compact Hausdorff space with $[0, 1)$?