TOPOLOGY PRELIMINARY EXAMINATION
Tuesday, January 2, 1996

Do Problems 1 through 4, plus your choice of any 4 problems in the ones numbered 5 through 12. All problems are weighted equally. Do each problem on a separate page. Include a cover sheet which lists the problems you have chosen.

Problem 1. Prove that every compact Hausdorff space is normal.

Problem 2. Prove that every compact metric space has a countable basis.

Problem 3. Prove that a complete metric space is not the union of countably many nowhere dense sets.

Problem 4. Show that the product of two connected spaces is connected.

Problem 5. Give an example of a Hausdorff space that is not regular.

Problem 6. Show that the arbitrary product of regular spaces is regular.

Problem 7. Suppose that X is a metric continuum and H is a closed subset of X. Show that there is a subcontinuum M of X containing H so that no proper subcontinuum of M contains H. [Definition: A *continuum* is a compact, connected space.]

Problem 8. Show that a linearly ordered space is compact iff every nonempty subset has both a supremum and an infimum.

Problem 9. Show that the first uncountable ordinal (with the usual order topology) is limit compact but not compact. [Definition: A space is *limit compact* iff every infinite set has a limit point.]

Problem 10. Let X be path connected, and $x_0 \in X$. Define the group operation for $\pi_1(X, x_0)$, and show that it is well-defined operation which satisfies the associative law. [You need not verify the other group properties.]

Problem 11. Let E and B be path connected spaces, let $p : E \to B$ be a covering map, and suppose that $p(e_0) = b_0$. Prove that every path in B which starts at b_0 lifts to a unique path in E which starts at e_0.

Problem 12. Let X be the quotient space which is obtained from the unit sphere by identifying antipodal points. Prove that X is not simply connected.