TOPOLOGY PRELIMINARY EXAMINATION
Saturday, May 24, 1997

1. (3 pts each) Complete the following statements:

   (a) The statement that the space X is completely regular (Tychonoff) means that _____

   (b) The statement that the space X is paracompact means that _____

   (c) The statement that the set H is connected means that _____

   (d) The statement that the set $A \subset B$ is a component of B means that _____

   (e) The statement that the space X is Lindelöf means that _____

   (f) The statement that the space X is separable means that _____

   (g) The statement that the the function $f : X \to Y$ is a quotient map means that _____

2. (10 pts each) Prove the following theoems:

   (a) If X is separable and $f : X \to Y$ is continuous, then Y is separable.

   (b) If X is a separable metric space, then X has a countable basis.

   (c) If X is a paracompact Hausdorff space, then X is regular.

   (d) If X is compact, then X is not the sum of countabley many nowhere dense subsets.

   (e) If $\mathcal{H}$ is a locally finite collection, then $\{cl(H)|H \in \mathcal{H}\}$ is locally finite.

   (f) If $\mathcal{H}$ is a collection of connected sets with a point in common, then $\cup \mathcal{H}$ is connected.

3. (10 pts each) Prove or disprove any three of the following:

   (a) A regular Lindelöf space is normal.

   (b) The product of two Lindelöf spaces is Lindelöf.

   (c) The product of a collection of compact spaces is compact.

   (d) If X is compact and Y is paracompact, then $X \times Y$ is paracompact.

   (e) A collecton of sets $\mathcal{H}$ is monotonic if it is true that if H and K are in $\mathcal{H}$, then either $H \subset K$ or $K \subset H$. The common part of a monotonic collection of compact connected sets is connected.

   (f) A quotient map is closed. (a continuous function is closed provided that the image of closed sets are closed.)

   (g) The space X is compact if and only if every collection of closed subsets with the finite intersection property has a common part.