Design Theory Prelim – 2004

1. Prove that a commutative idempotent latin square (quasigroup) of order n exists if and only if n is an odd integer.

2. Prove that there are at least 5 MOLS(102) (or equivalently an Orthogonal array OA(102,7)). Describe what ingredients you would use, and state how you know the ingredients exist.

3. Let G be an edge-colored copy of K_4 in which the edges colored 1 induce a copy of K_3 and the remaining edges are colored 2. A G-decomposition of $2K_n$ is a collection C of copies of G such that each pair of vertices in K_n is joined by an edge colored 1 in exactly one copy of G in C, and each pair of vertices in K_n is joined by an edge colored 2 in exactly one copy of G in C.

    a. Find a G-decomposition of K_7.
    b. Find a necessary condition for the existence of a G-decomposition of K_n that is sufficiently general that it shows that there is no G-decomposition of K_{257}.

4. a. Construct a projective plane of order 4.
    b. Does this contain an affine plane? Why or why not?

5. Show that:

    a. The number of idempotent MOLS(n) is at most n-2.
    b. If there exist k MOLS(n) then there exist k-1 idempotent MOLS(n).