<u>Design Theory</u>
<u>May, 2005</u>

1. Use the quasigroup given below and the Bose Construction to construct a Steiner triple system of order 15.

○	1	2	3	4	5
1	1	5	2	3	4
2	5	2	4	1	3
3	2	4	3	5	1
4	3	1	5	4	2
5	4	3	1	2	5

2. Let (S, T) be the triple system of order 25 constructed using the <u>SkolemConstruction</u> and the quasigroup

○	1	2	3	4	5	6	7	8
1	1	8	2	5	3	7	4	6
2	8	2	5	3	7	4	6	1
3	2	5	3	7	4	6	1	8
4	5	3	7	4	6	1	8	2
5	3	7	4	6	1	8	2	5
6	7	4	6	1	8	2	5	3
7	4	6	1	8	2	5	3	7
8	6	1	8	2	5	3	7	4

List <u>ALL</u> of the triples containing the symbol (5,2).
IT IS <u>NOT</u> NECESSARY TO CONSTRUCT (S, T) – just list the triples containing (5,2).

3. Give a solution to Heffter's Difference Problem for $n = 21$ as follows.
   a. Partition $\{1,2,3,4,5,6,7,8,9,10\}\setminus\{7\}$ into difference triples.
   b. Write out the base blocks (including the base block for the short orbit) for the cyclic triple system constructed from this solution.
   c. What is the triple containing
      i. The symbols 4 and 9
      ii. The symbols 5 and 12.

4. Let (K, T) be the Kirkman triple system of order 27 constructed from the PBD (P,B) given by:
   a. $P = \{1,2,3,4,5,6,7,8,9,10,11,12,13\}$, and
   b. $B = \{\{1,2,3,10\},\{4,5,6,10\},\{7,8,9,10\},\{1,5,9,12\},\{2,6,7,12\},\{3,4,8,12\},$
      $\{1,4,7,11\},\{2,5,8,11\},\{3,6,9,11\},\{1,6,8,13\},\{2,4,9,13\},\{3,5,7,13\},$
      $\{10,11,12,13\}\}$

   and the Kirkman Triple System

∞,x1,x2	∞,y1,y2	∞,z1,z2	∞,w1,w2
y1,z1,w1	x1,z1,w2	x1,w1,y2	x1,y1,z2
y2,w2,z2	x2,w1,z2	x2,y1,w2	x2,z1,y2

   where $x < y < z < w$.
   Construct the parallel class containing the triple $\{\infty,(6,1),(6,2)\}$.

5. Construct the finite field $(Z_2[x], +, \circ, 1+x+x^3)$.

6. Rename the elements in Question 5 as follows, and then construct the pair of orthogonal latin squares L(3) and L(6) (i.e., the latin squares determined by symbols 3 and 6 in the finite field construction).

Symbol in Question 5	Symbol in the latin square
0	8
1	1
x	2
x^2	3
$1 + x$	4
$x + x^2$	5
$1 + x + x^2$	6
$1 + x^2$	7