Design Theory Prelim
2018

1. A 5-cycle system of order n is a partition of the edges of K_n, each element of which induces a 5-cycle.
   a. Show that a necessary condition for the existence of a 5-cycle system of order n is that $n \equiv 1$ or 5 (mod 10).
   b. Find a cyclic 5-cycle system of order 11 using difference methods.
   c. A 5-cycle system is said to be resolvable if the set of 5-cycles can be partitioned into sets, called parallel classes, such that each vertex appears in exactly one 5-cycle in each parallel class.
      i. How many parallel classes does a resolvable 5-cycle system of order n contain?
      ii. How many 5-cycles are there in a parallel class of a resolvable 5-cycle system of order n?
      iii. What does this, together with 1(a), tell you about the existence of resolvable 5-cycle systems?

2. Find:
   a. 3 MOLS(4)
   b. A BIBD of order 16 in which all blocks have size 4 by using 2(a).

3. Let DPFF(n) be the maximum number of MOLS(n) that can be constructed by using just the Direct Product construction together with the Finite Field construction. (So, think of the MacNeish conjecture as asserting that the number of MOLS(n) is at most DPFF(n).)
   a. What is DPFF(63)?
   b. What is DPFF(n)? Substantiate your answer by indicating how the two constructions can be used to obtain this number of MOLS(n).
   c. Is the MacNeish Conjecture true or false? Why? Feel free to refer to something you constructed in MATH 6770, without giving details.

4. Prove that:
   a. There exists a symmetric idempotent quasigroup of order n if and only if n is odd.
   b. If there exist k MOLS(n) then there exist k-1 MOLS(n), each of which is idempotent.