

Motion of Charges in Combined Electric and Magnetic Fields; Measurement of the Ratio of the Electron Charge to the Electron Mass

Object: Understand the laws of force from electric and magnetic fields. You will measure the radius of the path of an electron in a magnetic field; you will demonstrate how to counteract that motion with an electric field; and you should understand the conditions (electric field E , magnetic field B , and electron velocity v) under which this happens.

Equipment: Evacuated glass bulb containing an electron gun (to produce a beam of electrons with controllable kinetic energy) and parallel plates to create an electric field perpendicular to the beam of electrons; pair of magnet coils; variable-voltage power supply for the electron beam; dual power supply to provide (1) current to the magnet coils, and (2) voltage to the parallel plates; multimeter to be used as **voltmeter** measure the voltage between the plates; multimeter to be used as ammeter (10 A) setting to measure current to magnet coils; wires with banana plug connectors.

Introduction

It has long been known that charged particles such as electrons and protons move in circular paths in a magnetic field. The Van Allen radiation belts in space around the earth consist of energetic electrons and protons trapped in the magnetic field of the earth. Magnetic fields are also used to guide the motion of protons and electrons in accelerators for research and medical purposes, and the orbiting motion of charges in a magnetic field is the basis for measuring the mass of atoms in a sample in order to identify what's in it (mass spectrometry).

A particle with charge q moving in an electric field E and a magnetic field B experiences a force F :

$$F = qE + qv \times B \quad (1)$$

where you've learned to determine the directions the electric and magnetic forces in class. In a vacuum in which collisions between particles are not very frequent, a particle with charge q , mass m , and velocity v perpendicular to a uniform magnetic field B (no E) moves in a circular path with radius

$$r = \frac{mv}{qB} \quad (2)$$

One can also deflect the trajectory of a charged particle with an electric field, although not into a circular path. If the electric force on the particle is set to be equal and opposite to the magnetic force, the net force on the particle will be zero. From Eq. (1), this will happen if

$$v = \frac{E}{B} \quad (3)$$

for the correct orientation of the fields (use the right hand rule). For this velocity only, the particle travels in a straight line in the combined electric and magnetic fields. So we can measure its velocity if we know the strength and orientation of the electric and magnetic fields.

How do we give velocity to a charged particle? We accelerate it through a known voltage difference V_A . If it starts from rest, it gains kinetic energy $K = \frac{1}{2} mv^2 = qV_A$ or a velocity

$$v = \sqrt{\frac{2qV_A}{m}} \quad (4)$$

Equating Eq. (3) and (4) and solving for q/m , we find

$$\frac{q}{m} = \frac{v^2}{2V_A} = \frac{1}{2V_A} \left(\frac{E}{B} \right)^2 \quad (5)$$

Why is this of interest? Even by the end of the 19th century, people didn't know either the mass or the charge of the electron, but were able to determine the ratio of e/m with an experiment like this one. As you perform this more modern experiment, you may appreciate some of the difficulties these scientific pioneers faced in trying to measure things they didn't yet understand with hand-made glass tubes, sealing wax, leaky batteries, etc. – all while dressed in coats and ties. More importantly, the same or similar technique can be used to measure the charge-to-mass ratio of any charged particle, and can thus be used to identify the chemical constituents of all kinds of mixtures (pollutants, explosives, food... you name it).

Equipment

The central piece of equipment is a nearly spherical glass tube that has been evacuated of air. The narrow neck of the tube is plugged into a socket through which electrical connections are made to conductors inside the tube. See Fig. 1 and Fig 2 for a drawing of the tube and a wire diagram.

Inside the narrow neck of the tube is a filament that is heated by a current passing through it. When it glows white hot, it emits electrons. When emitted, the electrons have very little kinetic energy (typically a few eV or less). Using a power supply with a variable DC voltage of 0 to 5000V, a voltage difference V_A is applied between the filament (also referred to as the “cathode”) and a metal plate with a slit opening in it (“anode”) located where the tube neck joins the spherical tube. This voltage accelerates the electrons toward the anode. They gain a velocity given by Eq. (4) by the time they hit the anode. Although most of the electrons will hit the anode, a small fraction of them instead go through the open slit into the spherical volume of the tube. Thus this electron gun generates a ribbon-shaped beam of electrons, with each electron having kinetic energy $K = eV_A$.

You can't see the electrons directly, but you can see where they hit the card inside the tube. The square card is coated with a chemical that emits light (“fluoresces”) when struck by energetic electrons (like a TV tube in the pre-flat screen days). As viewed from above the bulb, the card is angled slightly to intercept the ribbon of electrons at different points along its trajectory indicating where the beam of electrons is located. See Fig. 3.

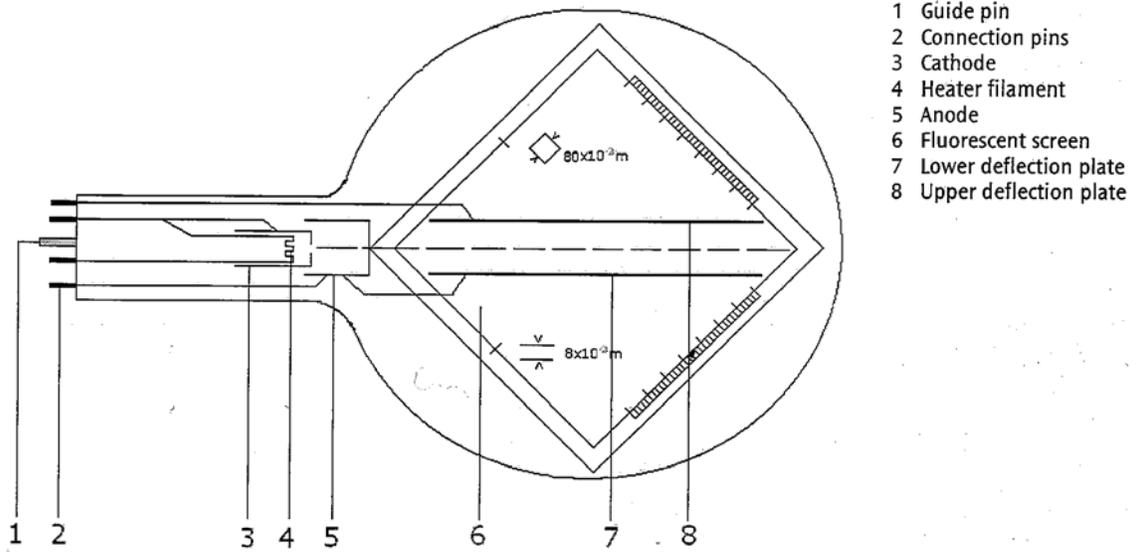


Figure 1⁽¹⁾

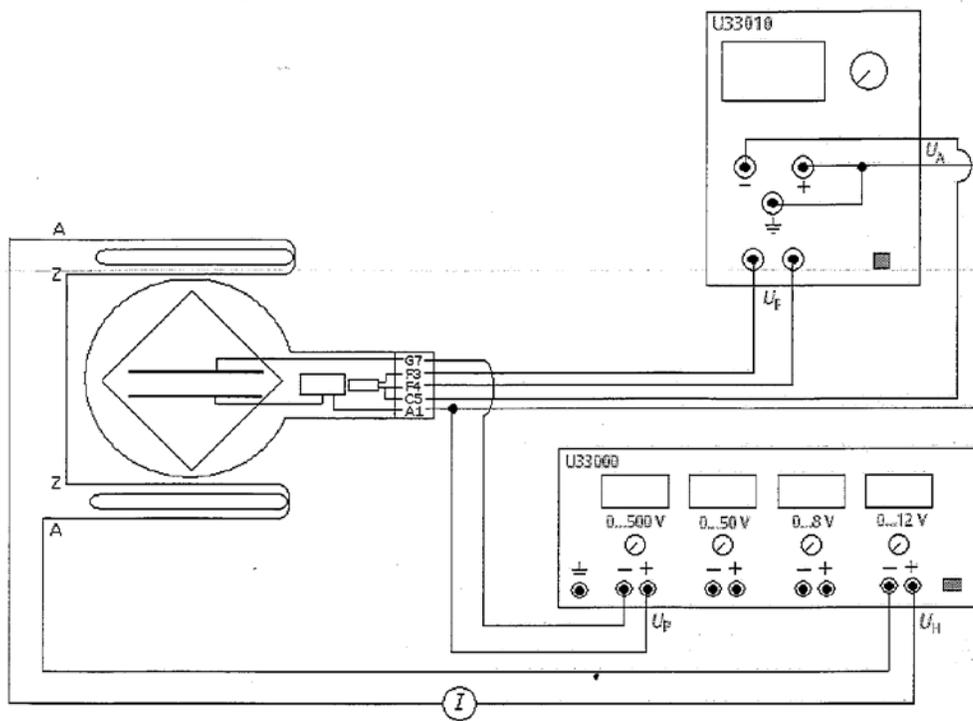


Figure 2⁽²⁾

Also inside the tube is a pair of parallel metal plates separated by a distance $\Delta y = 8$ mm (according to the specifications that came with the tube). The electron beam normally passes between this pair of parallel plates. When a voltage difference V_P is set between these parallel plates, it generates a perpendicular electric field E between the parallel plates. If the electric field can be assumed to be nearly uniform, you can calculate the strength of the electric field E if you know V_P and the distance between the parallel plates.

Lastly, there is a pair of circular magnet coils that bracket the spherical tube. With these coils connected in series, a current through the coils generates a horizontal magnetic field perpendicular to the direction of the beam. Each coil has N turns of wire, a radius a , and is also separated from its partner by the same distance a . This is the so-called Helmholtz configuration, which generates a **nearly** uniform magnetic field between them (more on this later).

We don't measure the magnetic field directly, but we **do** know that its strength is proportional to the current through the wire. For the Helmholtz configuration, it is fairly easy to calculate (albeit with a little more physics) the magnetic field between the coils to be

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N}{a} I_H \quad (6)$$

for a current I_H through the coils.

SAFETY

You will be using high voltage power supplies that are potentially lethal if you misuse them. This apparatus is safe and it would be very difficult to get a shock from this experiment. Nonetheless, I'm sure you could do it if you try, or are careless. So here are some rules to follow:

1. If you have to unplug wires from a power supply or what it is connected to, always turn OFF the power supply first. If there is any doubt in your mind, turn it off. You should not have to unplug any wires during this lab, so this will not likely be an issue for you.
2. The only things you can get a shock from are the exposed conductors on and within the banana plugs at the ends of the wires. So avoid touching them when the power supplies are on. It is perfectly safe to touch the tube, coils, and the knobs and switches of power supplies.
3. When working with electric circuits, it's a good idea to keep your hands dry. Wet skin has lower resistance than dry skin.

Procedure

1. Look over the apparatus to identify the pieces identified in the section above and compare with the diagrams in Fig. 1 and Fig. 2.
2. Turn on the multimeters you'll use. The first is used as an ammeter to measure I_H . The wires into it go to the "10 A" and "COM" receptacles on the multimeter, and the meter is set to the "10A" setting on the dial. The second is used as a voltmeter, and the meter is set on the 1000 V setting on the dial. The wires go to the "V Ω " and "COM" receptacles on the multimeter.
3. Turn on the small power supply that (1) supplies a fixed voltage to the filament, and (2) the variable accelerating voltage V_A . If the equipment is working properly, you will see a yellow-white glow in the neck of the tube, indicating the filament is on. When looking down at the tube from the top, the anode with the slit in it can be clearly seen.
4. Using the knob on the small power supply, turn up V_A until you see a clear blue line appear on the fluorescent card; this should happen around $V_A = 1.5$ kV. This light is made by the electrons in the beam striking the fluorescent card. The greater the beam energy, the more fluorescence you can see.

The beam should go straight across the card from one corner (vertex) of the card to the other. If you think that there is a misalignment of the beam so that it misses the opposite corner by more than a few mm, talk to your instructor.

Increase the accelerating voltage to $V_A = 2.0$ kV to give you a reasonably bright beam. You do not have a separate multimeter to measure V_A because the voltage is too high for standard multimeters. Instead, read the voltage from the analog meter on the front of the power supply; you should be able to read it to the nearest 100 V (0.1 kV).

5. The larger power supply provides a variable current $I_H = 0-1$ A to the Helmholtz coils (knob on far right) and variable voltage $V_P = 0 - 500$ V across the parallel plates (knob on far left). **Turn both of these knobs all the way down (counter-clockwise)**, turn on the power supply.

Leaving V_P set to zero, turn up the right knob that controls the current. If your set-up is correct, your ammeter will show an increase of current. Also, the electron beam will deflect into a path that looks circular (it doesn't complete a full circle) as shown by the solid curve in Fig. 3. The greater the curvature of the beam correlates to a smaller radius of the circular path.

Using the knob, vary the value of I_H and see how the radius changes. Compare what you see with your expectation from Eq. (2). Then leaving I_H fixed, slowly vary the accelerating voltage V_A between 2.0 kV and 4.0 kV. Again, compare what you see with your expectation from Eqs. (2) and (4).

Part I.

Now you will measure the radius r of the electron orbit for different values of B and v . It would seem straightforward to measure the radius of a circle, but, it is not that simple! First, the electron beam is inside a glass tube where you can't put a ruler wherever you want. Second, you can only see a portion of the circle made by the beam. However, you **can** see where the beam enters at one vertex of the card and where it crosses the scale (large ticks are 1 cm apart; small ticks 2 mm apart) on the side of the card. The distance along this side from the top vertex is labeled “ d ” in Fig. 3. The scale on the card begins at $d = 10$ mm and stops at $d = 70$ mm. One can use geometry to show that the radius can be found from d as follows:

$$r = \frac{s^2 + d^2}{\sqrt{2}(s - d)} \quad (7)$$

where s is the length of the side of the card.

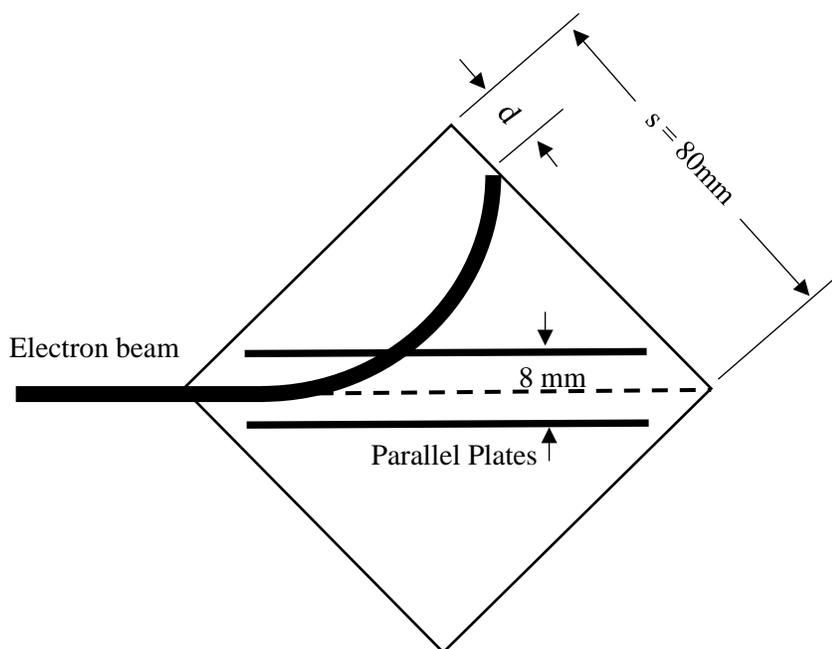


Figure 3

1. Set the accelerating voltage at the lowest value for which the blue electron beam can be seen (probably between 1.0 - 1.5 kV). Record this voltage on your datasheet. Be sure that V_P is set to zero for all of Part I. Increase the current I_H such that $d = 40$ mm. Record the current I_H for this value of d , then increase the current until $d = 30$ mm. After recording the current for $d = 30$ mm, determine the current necessary for d to equal 20 mm, 10 mm, and 0 mm (beam crosses top vertex of the card).

Set the accelerating voltage to 2.0 kV and repeat; then 3.0 kV, and 4.0 kV, recording your data of V_A , I_H , and d .

- Using each pair of d and I_H , calculate values for B and r from Eqs. (6) and (7).
- Using a spreadsheet program or a piece of graph paper, create a plot of r vs $\frac{m}{qB}$, and use Eq. (2) to determine a velocity that corresponds to each V_A . All four plots should be on the same graph.
- Why is the velocity of the electron different for each accelerating voltage? Use Eq. (4) to calculate the expected value of v for each V_A , and compare it with what you obtain from your plots. Calculate the percent difference between the two values of v for each setting of V_A .

Part II

- Reduce I_H to zero, and set V_A to 2.0 kV. Increase the parallel plate voltage to create an electric field between the parallel plates. Note how the beam deflects in the opposite direction. Again set V_P to zero.
- Set V_A to the lowest value you used in Part I. Increase I_H until you see a noticeable deflection of the beam. Then increase V_P until the beam goes “straight” through the gap between the parallel plates and intersects the far vertex of the card. You’ll probably note that the beam actually doesn’t go straight through, but first bends one way, then the other, then back again. Record the values of V_A , V_P , and I_H . Repeat for about 5 different values of I_H . Then set V_A to 2.0 kV and repeat; set V_A to 3.0 kV, then 4.0 kV, collecting up to 5 sets of data for each setting of V_A . **For the higher voltage settings, you will find that you cannot set V_P high enough to counteract the magnetic force for small values of d .**
- Using each pair of V_P and I_H , calculate E and B , and then on one graph, plot E vs. B for each setting of V_A . Using your plots and Eq. (3) determine v for each setting of V_A . Compare these values to your calculated values of v from Eq. (4) by calculating the percent difference between them.

Part III

We believe we know the mass and charge of the electron quite well these days, but that wasn’t true about 115 years ago when people weren’t yet sure electrons were identical particles. So this experiment confirmed that the ratio of the electron charge to the electron mass was the same for all electrons in the beam. How? By using Eq. (5). These days, we use this technique and similar ones to identify the mass of an unknown particle when we are fairly sure of its charge.

- For each of your data sets in Part II, calculate the value of e/m (electron charge-to-mass ratio) in units of C/kg. Average them, and compare them to what you’d calculate using the known values listed in your book. Calculate the percentage difference.

Part IV

1. Slowly return all of the knobs on the power supplies to 0V by turning the knobs counterclockwise.
2. Turn off both multimeters.
3. Turn off both power supplies.
4. Unplug the power cords of both power supplies from the AC outlets and return the power cords to the wall.

(1), (2) Figures 1 and 2 were copied from 3B Scientific Physics' "*Thomson Tube S U18555*", (2008).

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Name: _____

Banner ID: _____

Lab Group ID: _____

Number of Lab Partners: _____

Data Sheet

Motion of Charges in Combined Electric and Magnetic Fields; Measurement of the Ratio of the Electron Charge to the Electron Mass

Apparatus Constants	
Coil Radius (a)	68 (mm)
Number of Turns for Each Coil (N)	320 turns
Separation between the Upper and Lower Parallel Plates (Δy)	8×10^{-3} (m)
Length of the Side of the Fluorescent Card (s)	80 (mm)

Part I

$V_A =$ KV	
d (mm)	I_H (A)
40	
30	
20	
10	
0	

$V_A = 2.0$ KV	
d (mm)	I_H (A)
40	
30	
20	
10	
0	

$V_A = 3.0$ KV	
d (mm)	I_H (A)
40	
30	
20	
10	
0	

$V_A = 4.0$ KV	
d (mm)	I_H (A)
40	
30	
20	
10	
0	

Using each pair of d and I_H , calculate values for B and r from Eqs. (6) and (7). Using a spreadsheet program or a piece of graph paper, create a plot of r vs $\frac{m}{qB}$, and use Eq. (2) to determine a velocity that corresponds to each V_A . All four plots should be on the same graph. To compare the velocities you determined from your plots with theory, calculate the theoretical velocities using eqn. (4) and determine the % Difference.

Tabulate your results and attach to this data sheet. Although you may use Excel to make the repetitive calculations and to create your tables and graphs, you must provide at least one sample calculation for each of the physical characteristics ($v_{(4)}$, $B_{(6)}$, $r_{(7)}$, and the % Difference) that you have been requested to calculate.

Part II

$V_A =$ KV	
V_P (V)	I_H (A)

$V_A = 2.0$ KV	
V_P (V)	I_H (A)

$V_A = 3.0$ KV	
V_P (V)	I_H (A)

$V_A = 4.0$ KV	
V_P (V)	I_H (A)

Using each pair of V_P and I_H , calculate values for E and B . (Hint: the magnitude of the E field should be calculated using the physical parameters of the parallel plates in the bulb and the potential across these parallel plates.) Using a spreadsheet program or a piece of graph paper, create a plot of E vs B , and use Eq. (3) to determine a velocity for each V_A . All four plots should be on the same graph. To compare the velocities you determined from your plots with theory, calculate the theoretical velocities using eqn. (4) and determine the % Difference.

Tabulate your results and attach to this data sheet. Although you may use Excel to make the repetitive calculations and to create your table and graphs, you must provide at least one sample calculation for each of the physical characteristics (E , B , and the % Difference) that you have been requested to calculate.

Part III

For each of your data sets in Part II, calculate the value of e/m (electron charge-to-mass ratio) in units of C/kg. Average them, and compare the average to the known value listed in your book. Calculate the percentage difference.

Tabulate your results and attach to this data sheet. Again, you may use Excel, but sample calculations must be provided.

Part IV

Final Check List	
Return voltages to zero on both power supplies by turning the knobs counterclockwise.	
Turn off both Multimeters	
Turn off both Power Supplies	
Unplug the power cords of both power supplies from the AC outlets and return the power cords to the wall	

Questions:

1. Given that you know that these particles are negatively-charged electrons, what must be the direction of the magnetic field produced by the Helmholtz coils? Toward you or away from you?

What must be the direction of the current in the coils, clockwise or counterclockwise?

What must be the direction of the electric field you apply, up or down?

2. In Part I, you were requested to plot r vs $\frac{m}{qB}$ and to determine the velocities from the plots.

Briefly explain how determining the velocities from the plots would have been different if r vs $\frac{1}{B}$ had been plotted instead.

Briefly explain why plotting r vs B would be less desirable than plotting r vs $\frac{1}{B}$.

3. In Part II, you notice that the beam didn't go straight across the tube, but wobbled up and down. This probably results from the fact that the magnetic field produced by the Helmholtz coils is not quite uniform within the bulb. From the directions of the wobble and using what you learned from the deflection in Parts I and II, determine whether the actual magnetic field is stronger near the edge of the bulb compared to the center, or weaker. Explain your reasoning.

4. Errors:

It is likely you did not get perfect agreement between your measured and calculated values. List at least three reasons why you think there might be differences. For example, consider the possible complication of the earth's magnetic field (about 3×10^{-5} T) in Auburn. Based on your own measurements of the magnetic field strengths produced by your coils, could this lead to an "error" of the magnitude you measure?