I R A M P
10(1), June 2019, pp. 1-5

ISSN: 2229-39-44

Excitation of quantum linear oscillator by chirped laser pulse

V. A. ASTAPENKO AND E. V. SAKHNO

Moscow Institute of Physics and Technology (National Research University)

ABSTRACT: We theoretically investigated excitation of charged quantum linear oscillator by chirped laser pulse with the use of probability of the process during all time of pulse action. We consider the case of oscillator without relaxation and excitation from ground state. Calculations were made for arbitrary value of electric field strength with the use of exact expression for excitation probability. Dependence of excitation probability upon pulse parameters was analyzed numerically and using analytical formulas.

Let us consider the excitation of linear quantum oscillator by laser pulse from the ground state. For the probability of oscillator excitation from ground state during all time of pulse action the following expression is valid [1]:

$$W_{n0} = \frac{\bar{n}^n}{n!} \exp(-\bar{n}) \tag{1}$$

here $\bar{n}$ is average number of energy quanta at own frequency absorbed by oscillator during excitation. It is equal to (for the oscillator without relaxation):

$$\bar{n} = \frac{q^2}{2m\hbar\omega_0} \left|E(\omega_0)\right|^2 \tag{2}$$

here q, m, ω_0 are charge, mass and own frequency of oscillator. $E(\omega)$ is Fourier transform of electric field strength in the laser pulse. Further we consider pulse with Gaussian envelope and linear frequency chirp.

Fourier transform of electric field strength in the Gaussian pulse with linear frequency chirp has the form [2]:

$$E(\omega') = \frac{\sqrt{2\pi}\, E_0\, \tau}{\sqrt[4]{1+\alpha^2}} \exp\left\{-\frac{\omega'^2 + \omega^2 + 2i\,\alpha\,\omega'\,\omega}{\Delta\omega^2}\right\} \cos\left\{0.5\, arctg(\alpha) - \frac{\alpha\left(\omega'^2 + \omega^2\right) - 2i\,\omega'\,\omega}{\Delta\omega^2}\right\} \tag{3}$$

here E_0 is field amplitude, ω and τ are carrier frequency and duration of laser pulse, α is dimensionless chirp, $\Delta\omega$ is spectral width of the pulse which is equal to:

$$\Delta\omega = \frac{\sqrt{1+\alpha^2}}{\sqrt{2}\,\tau}\,. \tag{4}$$

In the resonance approximation $\left|\omega - \omega_0\right| << \omega_0$ one has

$$\left|E(\omega,\tau,E_0,\alpha)\right|^2 \cong \frac{\pi}{2}\frac{E_0^2\,\tau^2}{\sqrt{1+\alpha^2}}\exp\left\{-\frac{(\omega_0-\omega)^2\,\tau^2}{1+\alpha^2}\right\}. \tag{5}$$

Let us introduce the following dimensionless parameters:

$$\zeta = \sqrt[4]{1+\alpha^2}\,\frac{\Omega_{10}}{\omega_0},\ \beta = \frac{\omega_0\,\tau}{\sqrt{1+\alpha^2}},\ \Delta = \frac{\omega-\omega_0}{\omega_0}, \tag{6}$$

where

$$\Omega_{10} = \frac{d_{10}\,E_0}{\hbar} = \frac{q\,E_0}{\hbar\sqrt{2m\,\omega_0}} \tag{7}$$

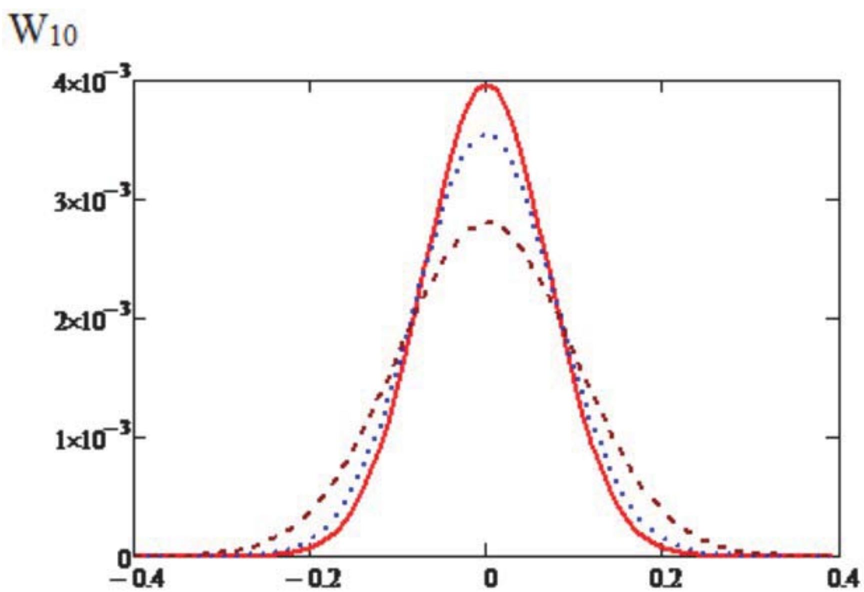
is resonance Rabi frequency and d_{10} is matrix element of electric dipole moment for the 0→1 transition in linear quantum oscillator. It is convenient for analytical description of oscillator excitation to express the average number of absorbed quanta $\overline{n}$ via dimensionless parameters in the form

$$\overline{n}(\Delta,\beta,\zeta) = \frac{\pi}{2}\zeta^2\beta^2\exp\left(-\beta^2\,\Delta^2\right) \tag{8}$$

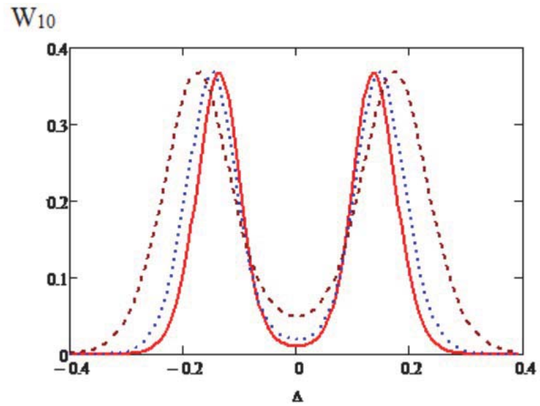
Here we used formulas (2), (5)-(7).

Substituting eq. (8) in eq. (1) we obtain the formula for numerical and analytical description of the excitation probability of quantum linear oscillator by chirped laser pulse from ground state.

The results of numerical calculations are presented in the figures below for the excitation probability of transition 0→1 in quantum oscillator for weak and strong field and various values of dimensionless frequency chirp. Calculations were made with the use of oscillator parameters q, m, ω_0 corresponding to vibration of CO molecule in harmonic approximation.



(a)



(b)

Fig.1. Spectrum of the excitation probability of transition 0⟶1 in quantum oscillator for weak field – E_0=10^{-3} a.u. (a), strong field - E_0=0.04 a.u. (b) and different values of dimensionless frequency chirp: solid line - α=0, dotted line - α=0.5, dashed line - α=1

Let us consider analytically the spectral dependence of the excitation probability of transitions 0⟶*n* in quantum oscillator. It is easy to obtain the position of spectral maxima using formulas (1) and (8). In weak field regime when

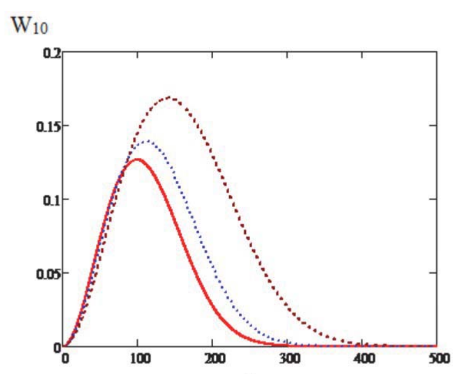
$$\Omega_{10}\,\tau < \sqrt{\frac{2n}{\pi}}\sqrt[4]{1+\alpha^2} \tag{9}$$

there is only one maximum at $\Delta = 0$ (see Fig.1 (a)). With increasing electric field strength (i.e. Rabi frequency Ω_{10}), this maximum becomes a minimum. When the inverse to (9) inequality holds, two maxima appear at the following detunings of the carrier frequency from the own oscillator frequency (according to Fig.1 (b)):

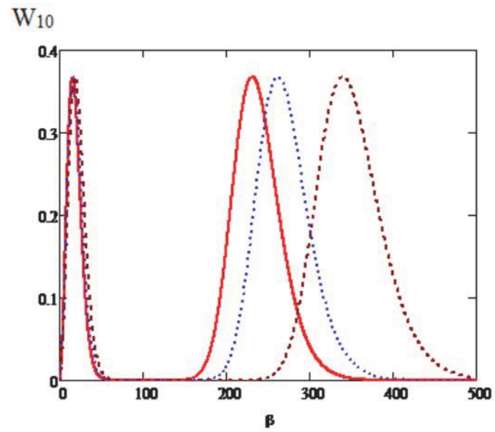
$$\left|\Delta_{1,2}\right| = \frac{\sqrt{1+\alpha^2}}{\omega_0\,\tau}\sqrt{\ln\left(\frac{\pi}{2\,n}\frac{\Omega_{10}^2\,\tau^2}{\sqrt{1+\alpha^2}}\right)}. \tag{10}$$

One can see from this formula that spectral distance between maxima in strong field regime grows with the increase of chirp modulus and amplitude of the field.

Figure 2 demonstrates the dependence of oscillator excitation probability at transition 0⟶1 as a function of dimensionless pulse duration (parameter β) for weak (a) and strong (b) field and for different values of frequency chirp.



(a)



(b)

Fig.2. Excitation probability of transition 0→1 **in quantum oscillator as function of dimensionless pulse duration (β) for weak field – E_0=10^{-3} a.u. (a), strong field - E_0=0.04 a.u. (b) and different values of dimensionless chirp: solid line - α=0, dotted line - α=0.5, dashed line - α=1**

As can be seen from Figure 2, with an increase in the field amplitude, one maximum becomes a minimum, and two new maxima appear, the distance between which increases with increasing magnitude of the chirp and the field amplitude.

For weak field amplitude when the following inequality holds (here *e* is the base of the natural logarithm)

$$\Omega_{10} < \sqrt{\frac{2\,n\,e}{\pi}\,\frac{|\omega-\omega_0|}{\sqrt[4]{1+\alpha^2}}} \tag{11}$$

we have

$$\tau_{\max} = \frac{\sqrt{1+\alpha^2}}{|\omega - \omega_0|} \tag{12}$$

For strong fields when inverse to (11) inequality holds only an approximate analytical description of these maxima is possible. Then one can obtain the following relations:

$$\tau_{\max,1} \cong \sqrt{\frac{2\,n}{\pi}\,\frac{\sqrt[4]{1+\alpha^2}}{\Omega_{10}}}\,,\quad \tau_{\max,2} \cong \frac{\sqrt{1+\alpha^2}}{|\omega - \omega_0|}\sqrt{2.8\ln\left(\sqrt{\frac{\pi}{2n}}\,\frac{\sqrt[4]{1+\alpha^2}\,\Omega_{10}}{|\omega-\omega_0|}\right)}. \tag{13}$$

The resonance case (Δ=0) should be treated separately and the result is:

$$\tau_{\max} = \sqrt{\frac{2n}{\pi}\,\frac{\sqrt[4]{1+\alpha^2}}{\Omega_{10}}}\,. \tag{14}$$

In conclusion, we give an expression for the electric field strength amplitude corresponding to the maximum probability of the excitation of transition 0→*n* with other fixed parameters:

$$E_{0\max} = \frac{\hbar\omega_0}{d_{10}}\sqrt{\frac{2n}{\pi}}\frac{\sqrt{1+\alpha^2}}{\omega_0\tau}\exp\left\{\frac{(\omega-\omega_0)^2\tau^2}{2(1+\alpha^2)}\right\}. \qquad (15)$$

Acknowledgments

The research was supported by the government order of the Ministry of Education and Science of the Russian Federation (Project No. 3.9890.2017/8.9).

References

[1] K. Husimi, Miscellanea in elementary quantum mechanics, II. // Progress of Theoretical Physics 9 381- 402 (1953).

[2] V. Astapenko *Interaction of ultrafast electromagnetic pulses with matter* Springer Briefs in Physics. Heidelberg, New York, Dordrecht, London. 2013. – 94 p.