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Analytical Results for the Contribution to the Stark Width of Hydrogenlike Spectral Lines due to the Monopole Interaction

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ABSTRACT: We use the formalism from paper by Mejri, Nguyen, and Ben Lakhdar (Europ. Phys. J. D 4 (1998) 125) to study analytically the monopole contribution to the width of hydrogenlike spectral lines. We show that the monopole contribution to the width has a non-monotonic dependence on the velocity of perturbing electrons. Namely, at relatively small electron velocities, as velocity increases, the width decreases. Then it reaches a minimum and (at relatively large electron velocities), as velocity further increases, the width increases. The non-monotonic dependence of the monopole contribution to the width on the electron velocity is a counter-intuitive result. The outcome that at relatively large electron velocities, the monopole contribution to the width increases with the increase of the electron velocity is in a striking distinction to the dipole contribution to the width, which decreases as the electron velocity increases. We show that in the situation, encountered in various areas of plasma research, where there is a relativistic electron beam (REB) in a plasma, the monopole contribution to the width due to the REB exceeds the corresponding dipole contribution by four orders of magnitude and practically determines the entire Stark width of hydrogenic spectral lines due to the REB.

Key words: Stark broadening; Stark width; monopole contribution

1. INTRODUCTION

The theory of the Stark broadening of hydrogenlike spectral lines by plasma electrons was initially developed by Griem and Shen [1] (later being presented also in books [2, 3]). It is usually called the Conventional Theory (hereafter CT), sometimes referred also as the standard theory. In the CT it was assumed that the motion of the perturbing electron can be described in frames of a two-body problem: the perturbing electron moves along a hyperbolic trajectory around a "particle" of the charge Z – 1 (in atomic units).

In paper [4] there was taken into account that actually it is a three-body problem: the perturbing electron, the nucleus, and the bound electron, so that trajectories of the perturbing electrons are more complicated. The authors of paper [4] showed analytically by examples of the electron broadening of the Lyman lines of He II that this effect increases with the growth of the electron density N_e, becomes significant already at $N_e \sim 10^{17}$ cm^{-3} and very significant at higher densities.

Analytical advances beyond the CT include, e.g., the development of the so-called generalized theory of the Stark broadening of hydrogenlike spectral lines by plasma electrons [5]. Details can be found also in books [6, 7] and references therein.

In all of the above works, the authors focused at the *dipole* interaction of the radiating ion with perturbing electrons. In distinction, in paper [8] the authors studied analytically the *shift* of hydrogenlike spectral lines due to the *monopole* interaction with plasma electrons.

In the present paper we use the formalism from paper [8] to study analytically the monopole contribution to the width of hydrogenlike spectral lines. We show that the monopole contribution to the *width* has a non-monotonic dependence on the velocity of perturbing electrons. Namely, at relatively small electron velocities, as B increases, the width decreases. Then it reaches a minimum and (at relatively large electron velocities), as B further increases, the width increases.

2. ANALYTICAL RESULTS

The monopole interaction potential can be represented as follows (Eq. (3b) from paper [8])

$$V^{(0)}(t) = -e^2[1/|R(t) - 1/r]\ E[R(t) < r], \tag{1}$$

where R and r are the absolute values of the radii-vectors of the perturbing electron and of the bound electron, respectively; E[…] is the Heaviside function manifesting the fact that the monopole interaction vanishes for R(t) > r. According to Eq. (17) from paper [8], for the Lyman lines, the monopole contribution to the shift, caused by N_e electrons/cm^3 of velocity v, is given by

$$d_{nl\to 1s} = 2\pi N_e v \int_0^{\rho_{max}} \rho \sin[\langle nl|\Phi_0|nl\rangle - \langle 1s|\Phi_0|1s\rangle]\, d\rho, \tag{2}$$

where the matrix elements of the electron broadening operator have the form

$$\langle nlm|\Phi_0|nl`m`\rangle = -[e^2/(\hbar v)](1 + u_0)\{\ln[(1+x)/(1-x)] - 2x\}\ E[R(t) < r_{nl}]\ \delta_{ll`}\delta_{mm}\ . \tag{3}$$

Here

$$x = [1 - (u^2 + u_0^2)/(1 + u_0)^2]^{1/2}, \quad u = \rho/r_{nl}, \quad u_0 = \rho_0/r_{nl}. \tag{4}$$

In Eq. (4), ρ is the impact parameter and

$$\rho_0 = (Z - 1)e^2/(m_e v^2), \qquad r_{nl} = (\langle nl|r^2|nl\rangle)^{1/2} = (a_0 n/Z)\{[5n^2 + 1 - 3l(l + 1)]/2\}^{1/2}, \tag{5}$$

where r_{nl} is the root-mean-square size of the radiating ion in the state of the quantum numbers *n* and *l*, a_0 is the Bohr radius. Here are some useful practical formulas from paper [8]:

$$e^2/(\hbar v) = [13.605/kT_e(eV)]^{1/2}, \qquad u_0 = [Z(Z - 1)/n^2]\ [13.605/kT_e(eV)]. \tag{6}$$

As noted in paper [8], from the condition $R < r_{nl}$ it follows that

$$u_{max} = \rho_{max}/r_{nl} = (1 + 2u_0)^{1/2}. \tag{7}$$

Equation (7) is equivalent to

$$\rho_{max} = (r_{nl}^2 + 2\, r_{nl}\rho_0)^{1/2}. \tag{8}$$

In paper [8] it was noted that for relatively high temperatures, such that $e^2/(\hbar v) << 1$, one has $|\langle nlm|\Phi_0|nlm\rangle| < 1$ because. In the opposite limit of relatively low temperatures, such that $u_0 >> 1$, the authors of paper [8] estimated that $|\langle nlm|\Phi_0|nlm\rangle|$ does not exceed $(4/3)2^{1/2}n/[Z(Z-1)]^{1/2}$. Then by limiting themselves to the range of parameters where Z is no less than 5 and *n* is no more than 4, the authors of paper [8] replaced in Eq. (2) sin[…] by its argument.

In the present paper we are interested in the monopole contribution to the width $w^{(0)}$. For the Lyman lines it can be represented by Eq. (2) with sin[…] replaced by cos[…]:

$$w^{(0)}_{nl\to 1s} = 2\pi N_e v \int_0^{\rho_{max}} \rho \cos[\langle nl|\Phi_0|nl\rangle - \langle 1s|\Phi_0|1s\rangle]\, d\rho, \tag{9}$$

In distinction to paper [8] we focus at the situation where n >> 1 (or practically n > 4), so that the contribution of the ground level can be disregarded and Eq. (9) simplifies to

$$w^{(0)}_{nl\to 1s} = 2\pi N_e v \int_0^{\rho_{max}} \rho \cos(<nl|\Phi_0|nl>)\, d\rho. \qquad (10)$$

In a further distinction to paper [8], we do not limit ourselves by the case where $|<nlm|\Phi_0|nlm>| < 1$. Therefore we keep the corresponding trigonometric function (cos [..]) in the integrand in Eq. (10).

$$w^{(0)}_{nl\to 1s} = 2\pi N_e v r_{nl}^2 (1 + u_0)^2 \int_0^{y} x \cos\{[e^2(1 + u_0)/(\hbar v)][\ln((1+x)/(1-x)) - 2x]\}, \qquad (11)$$

where

$$y = (1 + 2u_0)^{1/2}/(1 + u_0). \qquad (12)$$

We denote

$$A = (Z - 1)a_B/r_{nl}, \qquad B = \hbar v/e^2, \qquad (13)$$

B being the scaled dimensionless velocity of the perturbing electrons. Then the width $w^{(0)}_{nl\to 1s}$ can be represented in the following final form

$$w^{(0)}_{nl\to 1s} = (2\pi N_e r_{nl}^2 e^2/\hbar)\, F[A, B], \qquad (14)$$

where

$$F[A, B] = B(1 + A/B^2)^2 \int_0^{y(A, B)} x \cos\{(1/B + A/B^3)\, [\ln((1+x)/(1-x)) - 2x]\}\, dx. \qquad (15)$$

The upper limit of the integration in Eq. (15) is

$$y(A, B) = (1 + 2\, A/B^2)^{1/2}/(1 + A/B^2). \qquad (16)$$

Thus, the dependence of the width $w^{(0)}_{nl\to 1s}$ on the scaled dimensionless electron velocity B is given by the function F(A, B). Figure 1 shows a three-dimensional plot of this function.

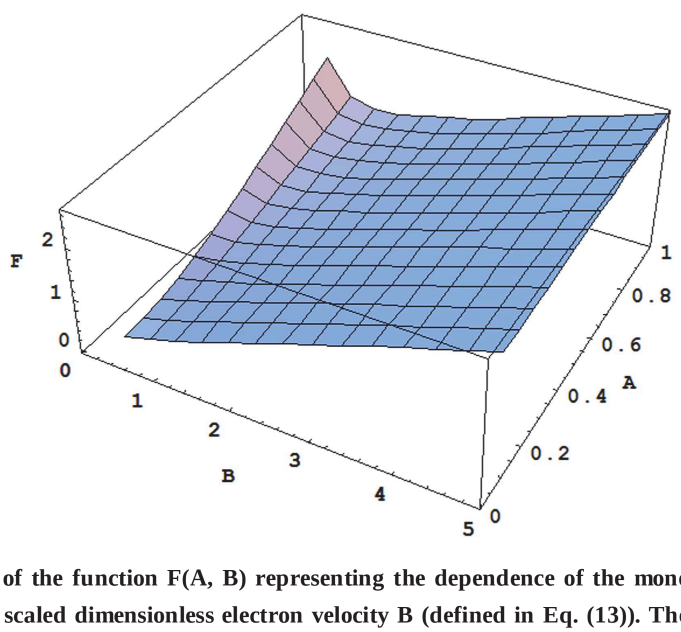

Fig. 1. Three-dimensional plot of the function F(A, B) representing the dependence of the monopole contribution to the width $w^{(0)}_{nl\to 1s}$ (from Eq. (14) on the scaled dimensionless electron velocity B (defined in Eq. (13)). The function F(A, B) is defined by Eqs. (15), (16).

Figure 2 presents the dependence of the function F(A, B) on the scaled dimensionless electron velocity B for three values of the parameter A: A = 1 (solid line), A = 0.5 (dashed line), and A = 0.25 (dash-dotted line). Both from Fig. 1 and Fig. 2 it is seen that the width $w^{(0)}_{nl\to 1s}$ has a non-monotonic dependence on B. Namely, at relatively small electron velocities, as B increases, the width decreases. Then it reaches a minimum and (at relatively large electron velocities), as B further increases, the width increases. The non-monotonic dependence of the monopole contribution to the width on the electron velocity is a *counter-intuitive result*.

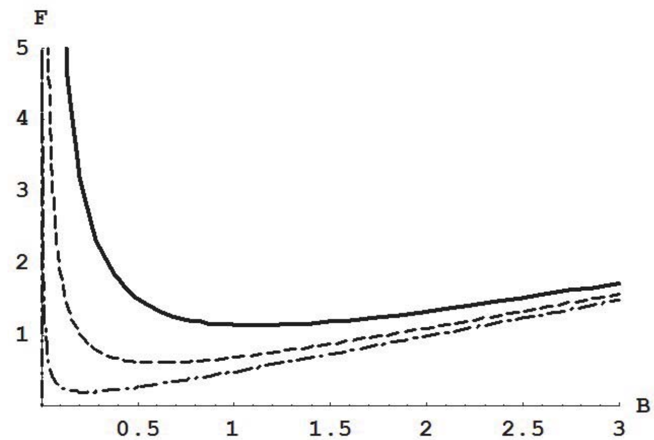

Fig. 2. Plot of the function F(A, B) representing the dependence of the monopole contribution to the width $w^{(0)}_{nl\to 1s}$ (from Eq. (14) on the scaled dimensionless electron velocity B (defined in Eq. (13)) for three values of the parameter A: A = 1 (solid line), A = 0.5 (dashed line), and A = 0.25 (dash-dotted line). The function F(A, B) is defined by Eqs. (15), (16).

For relatively large electron velocities, i.e., when B >> max(A, 1), the integration in Eq. (15) becomes trivial and we get F(A, B) = B/2, so that

$$w^{(0)}_{nl\to 1s} = \pi r_{nl}^2 N_e v. \tag{17}$$

Physically this means that the corresponding optical cross-section, i.e., the cross-section for the line broadening collisions, becomes equal to the “geometrical” cross section πr_{nl}^2.

It is remarkable that at relatively large electron velocities, the monopole contribution to the width increases with increasing velocity. This is in a *striking distinction* to the dipole contribution to the width, which decreases as the electron velocity increases.

There are situations, encountered in various areas of plasma research, where there is a relativistic electron beam (REB) in a plasma. In paper [15] the authors calculated analytically the *dipole* contribution w_d to the Stark width of hydrogenic spectral lines due to a REB. Based on the results of paper [15], the ratio of the corresponding *monopole* contribution w_m due to the REB to w_d can be estimated as follows:

$$w_m/w_d \sim (\hbar c/e^2)^2 \sim 10^4 >> 1. \tag{18}$$

It shows that the monopole contribution to the width due to the REB exceeds the corresponding dipole contribution by four orders of magnitude and practically determines the entire Stark width of hydrogenic spectral lines due to the REB.

3. CONCLUSIONS

We used the formalism from paper [8] to study analytically the monopole contribution to the width of hydrogenlike spectral lines. We showed that the monopole contribution to the width has a non-monotonic dependence on the velocity of perturbing electrons. Namely, at relatively small electron velocities, as velocity increases, the width decreases. Then it reaches a minimum and (at relatively large electron velocities), as velocity further increases, the width increases. The non-monotonic dependence of the monopole contribution to the width on the electron velocity is a *counter-intuitive* *result*.

We demonstrated analytically that at relatively large electron velocities, the so-called optical cross-section, i.e., the cross-section for the line broadening collisions, becomes equal to the "geometrical" cross section. Finally we emphasized that at relatively large electron velocities, the monopole contribution to the width increases with increasing velocity. This is in a *striking distinction* to the dipole contribution to the width, which decreases as the electron velocity increases.

Finally we discussed the situation (encountered in various areas of plasma research) where there is a relativistic electron beam (REB) in a plasma, and showed that the monopole contribution to the width due to the REB exceeds the corresponding dipole contribution by four orders of magnitude and practically determines the entire Stark width of hydrogenic spectral lines due to the REB.

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