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Excitation of a quantum harmonic oscillator under the action of pulses with different types of envelopes

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ABSTRACT: We investigate the features of excitation of a quantum harmonic oscillator under the action of pulses with different types of envelopes: with a sharp turn-on (exponential pulse) and a smooth doubly exponential pulse. Simple analytical expressions are obtained that describe the maxima of the probability of excitation of the oscillator by these pulses as a function of duration and carrier frequency. Differences in these dependences for two types of pulses are established.

The quantum harmonic oscillator is an important physical model that allows an analytical description for an arbitrary strength of an external perturbation. Significant advances in the technology of generating laser pulses with given parameters [1] make a detailed theoretical study of the interaction of such pulses with a quantum oscillator relevant. This article is devoted to revealing the specific features of the excitation of a quantum oscillator under the action of two types of pulses that differ in envelopes, namely, a pulse with a sharp turn-on and a smooth pulse.

Excitation by a pulse with a sharp turn-on will be considered using the example of an exponential pulse, which is described by the following time dependence

$$f_{\exp}(t,\omega_c,\tau)=\theta(t)\exp(-t/\tau)\cos(\omega_c\, t) \tag{1}$$

As a smooth pulse, we use a double exponential pulse of the following form

$$f_{2\exp}(t,\omega_c,\tau)=\exp(-|t|/\tau)\cos(\omega_c\, t) \tag{2}$$

ω_c , τ are carrier frequency and duration of the pulse.

Graphs of functions (1) and (2) are presented in Fig.1.

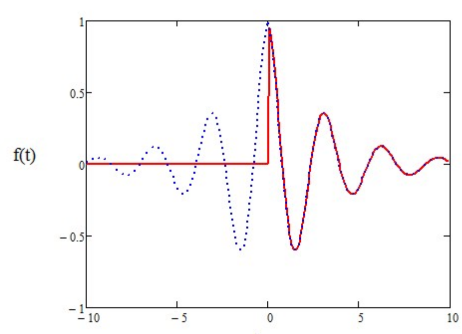

Fig.1. Two types of exciting pulses: the solid curve is the exponential pulse, the dotted line is the double exponential pulse; $\omega_c = 2$, $\tau = 3$

Next, we consider the excitation of a quantum oscillator from the ground state $\left|0\right\rangle$ to excited stationary state $\left|n\right\rangle$. The expression for the probability of this process is given by [2]

$$W_{n0} = \frac{\overline{n}^n}{n!}\exp\left(-\overline{n}\right) \tag{3}$$

here

$$\overline{n} = \Omega_0^2 \left|F\left(\omega_0, \omega_c, \tau\right)\right|^2 \tag{4}$$

is average number of oscillator quanta as a result of excitation, $F\left(\omega_0, \omega_c, \tau\right)$ – Fourier transform of the time dependence of the electric field in a pulse, ω_0 is eigenfrequency of oscillator, Ω_0 is the Rabi frequency, which is a measure of the strength of the perturbation of the oscillator.

For an exponential pulse, the Fourier transform of the field is

$$\overline{n}_{\exp} = \frac{1}{4}\frac{\Omega_0^2\,\tau^2}{1+\left(\omega_0-\omega_c\right)^2\tau^2} \tag{5}$$

and for double exponential pulse [3]

$$\overline{n}_{2\exp} = \frac{1}{4}\frac{\Omega_0^2\,\tau^2}{\left[1+\left(\omega_0-\omega_c\right)^2\tau^2\right]^2}\,. \tag{6}$$

Note that formulas (5) and (6) are valid in the rotating wave approximation, which is applicable for multicycle pulses $\omega_c\,\tau >> 1$, which is assumed.

Substituting formulas (5) and (6) into expressions (4) and (3) makes it possible to calculate the dependence of the probability of excitation of the oscillator from the ground to various excited stationary states as a function of the pulse parameters: duration, carrier frequency, and Rabi frequency for the two considered types of pulse envelope.

DEPENDENCE OF EXCITATION PROBABILITY OF A QUANTUM OSCILLATOR ON THE PULSE DURATION

For pulses (1) and (2), expressions for the extrema of these functions can be easily obtained in an analytical form. Thus, in the case of an exponential pulse, for the maximum in the dependence of the excitation probability on the pulse duration, we have

$$\tau_{\max}^{(\exp)} = \frac{1}{\sqrt{\Omega_0^2/4n-\left(\omega_c-\omega_0\right)^2}} \tag{7}$$

It follows that, when excited by an exponential pulse, the function $W_{n0}\left(\tau\right)$ can have only one maximum when the condition

$$\Omega_0 > 4\sqrt{n}\left|\omega_0-\omega_c\right| \tag{8}$$

is satisfied. When the inequality inverse to (8) is satisfied, there is no maximum, and the function $W_{n0}\left(\tau\right)$ is monotonically increasing.

The dependence of the probability of excitation of a quantum oscillator by an exponential pulse (1) at the 0→1 transition on the pulse duration is shown in Fig. 2 for various values of the Rabi frequency

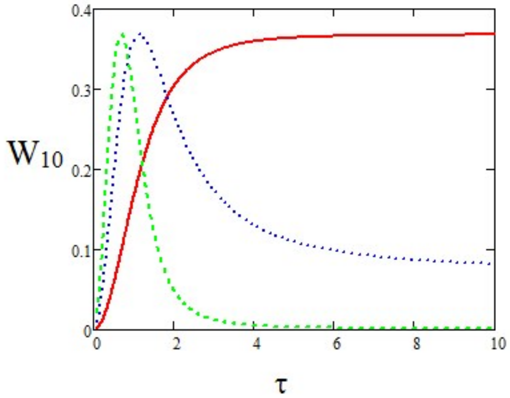

Fig.2. Dependence of the probability of excitation of a quantum oscillator by an exponential pulse (1) at the 0→1 transition on the pulse duration for different Rabi frequencies: solid curve - Ω_0=1, dotted line - Ω_0=2, dashed curve - Ω_0=3; ω_0=5, ω_c =5.5

When a quantum oscillator is excited by a double exponential pulse (2) in the case of a weak perturbation

$$\Omega_0 < 4\sqrt{n}\left|\omega_0 - \omega_c\right| \tag{9}$$

there is one maximum at the pulse duration

$$\tau_{\max}^{(2\exp)} = \frac{1}{\left|\omega_c - \omega_0\right|}. \tag{10}$$

Note that this maximum disappears in the case of a resonant carrier frequency of the pulse, when $\omega_c = \omega_0$. For higher Rabi frequencies (strong perturbation)

$$\Omega_0 > 4\sqrt{n}\left|\omega_0 - \omega_c\right| \tag{11}$$

the maximum in the τ-dependence, the position of which is determined by formula (10), turns into a minimum, and in this case, two other maxima appear at pulse durations equal to

$$\tau_{\max,1,2}^{(2\exp)} = \frac{\Omega_0 \pm \sqrt{\Omega_0^2 - 16n\left|\omega_c - \omega_0\right|^2}}{4\sqrt{n}\left|\omega_c - \omega_0\right|^2}. \tag{12}$$

The dependence of the probability of excitation of a quantum oscillator by a double exponential pulse (2) at the 0→1 transition on the pulse duration is shown in Fig. 3 for various values of the Rabi frequency.

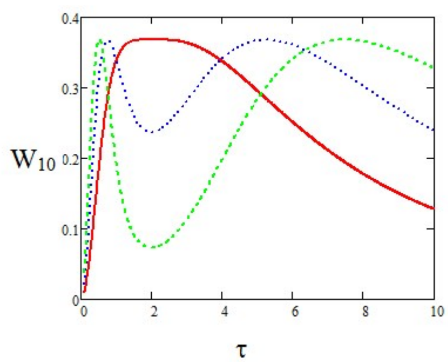

Fig.3. Dependence of the probability of excitation of a quantum oscillator by a double exponential pulse (2) at the 0→1 transition on the pulse duration for different Rabi frequencies: solid curve - Ω_0=2, dotted line - Ω_0=3, dashed curve -Ω_0=4; ω_0=5, ω_c =5.5

SPECTRAL DEPENDENCE OF THE PROBABILITY OF EXCITATION OF A QUANTUM OSCILLATOR

Exponential pulse

Using formulas (3) - (5), one can obtain the following expression for the maxima of this spectral dependence

$$\omega_{\max}^{(\exp)} = \omega_0 \pm \sqrt{\Omega_0^2/4n - \tau^{-2}}\,. \tag{13}$$

These maxima are realized if

$$\Omega_0 > \frac{2\sqrt{n}}{\tau}. \tag{14}$$

Otherwise, there is one maximum at the natural frequency of the oscillator $\omega_c = \omega_0$.

It follows from formula (13) that in the limit of long pulses ($\tau \to \infty$):

$$\omega_{\max}^{(\exp)} = \omega_0 \pm \Omega_0 / 2\sqrt{n}\,, \tag{15}$$

i.e. the difference between the frequencies of the maxima in the case of an exponential pulse is equal to

$$\Delta\omega_{\max}^{(\exp)}(\tau \to \infty) = \frac{\Omega_0}{\sqrt{n}} \neq 0 \tag{16}$$

This is illustrated in Fig.4.

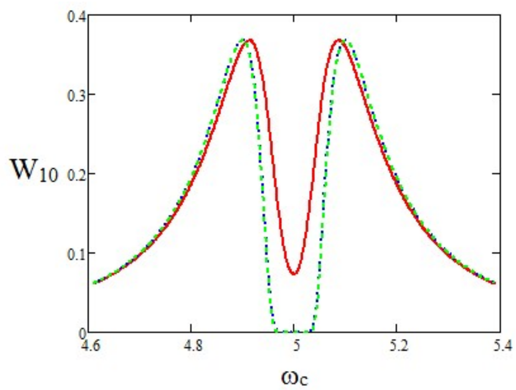

Fig.4. Excitation spectrum of a quantum oscillator by an exponential pulse with different duration at the 0→1 transition: solid curve - τ=20, dotted line - τ=100, dashed curve - τ=1000, Ω_0=0.2, ω_0=5

As can be seen from the figure, in the case under consideration, the spectral width of the maxima practically does not change with an increase in the duration of the exponential pulse.

Double exponential pulse

Formulas (3), (4), (6) give for the values of the spectral maxima upon excitation of a quantum oscillator by a double exponential pulse

$$\omega_{\max}^{(2\exp)} = \omega_0 \pm \frac{1}{\tau}\sqrt{\frac{\Omega_0\,\tau}{2\sqrt{n}} - 1}\,. \tag{17}$$

As follows from (17), two maxima appear if

$$\Omega_0 > \frac{2\sqrt{n}}{\tau}. \tag{18}$$

Otherwise, there is one maximum at the frequency $\omega_c = \omega_0$.

Note that inequalities (14) and (18) differ insignificantly. At the same time, in contrast to the limiting case (16), formula (17) gives

$$\Delta\omega_{\max}^{(2\exp)}(\tau \to \infty) \to \sqrt{\frac{2\Omega_0}{\sqrt{n}\,\tau}} \to 0. \tag{19}$$

Thus, in the limit of long pulses with a double exponential envelope, the difference in the frequencies of the maxima tends to zero. In this case, the frequency of the emerging maximum coincides with the eigenfrequency of the oscillator. The above is shown in Fig.5.

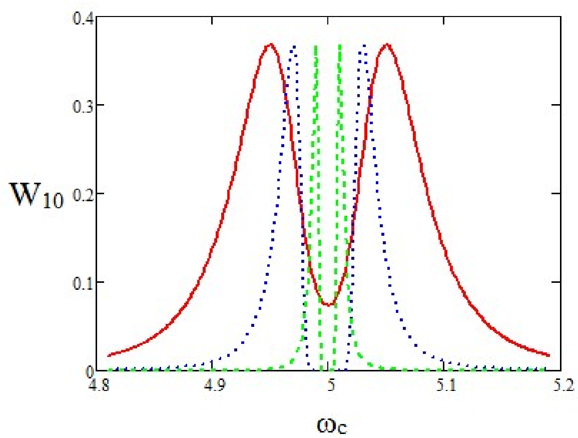

Fig.5. Excitation spectrum of a quantum oscillator by a double exponential pulse with different duration at the 0→1 transition: solid curve - τ=20, dotted line - τ=100, dashed curve - τ=1000, Ω_0=0.2, ω_0=5

It also follows from the figure that the width of the spectral maxima decreases with increasing duration of the double exponential pulse. This is in contrast to the case of excitation of the oscillator by an exponential pulse when the width of the spectral maxima remains constant in the limit of long pulses.

CONCLUSIONS

When a quantum oscillator is excited at transitions between stationary states 0→n by a pulse with a sharp turn on (exponential pulse), the probability of excitation upon a weak perturbation is a monotonically increasing function of the pulse duration. As the perturbation strength increases, this function has only one maximum, the position of which is determined by formula (7).

When oscillator is excited by a "smooth" pulse (double exponential pulse), in the case of a weak perturbation, the function $W_{n0}(\tau)$ has one maximum (10). As the perturbation strength increases, this maximum transforms into a minimum, and two maxima appear (12).

The number of spectral maxima when the quantum oscillator is excited by pulses with different envelopes is the same. The difference arises in the limit of long pulses. Then, in the case of pulses with a sharp turn-on, the frequencies of the maxima differ by an amount proportional to the Rabi frequency. In the same limit for "smooth" pulses, the difference between the frequencies of the two maxima tends to zero. In addition, in the first case, the width of the spectral maxima practically does not change at large values of pulse durations, and in the second case, this width decreases.

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