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# Two approaches to generalizing the theory of pulsed excitation of matter from micro to macro description

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**ABSTRACT:** We discuss two approaches to generalizing the description of pulsed excitation of matter from the micro to the macro level. The first of them was proposed by the author earlier in article [1]. The second approach is based on the use of the Lorentz — Lorenz relation to describe the relationship between the dynamic polarizability of atoms of a medium and its dielectric constant. Different limiting cases from the second approach are considered and corresponding expression for energy transferred from electromagnetic pulse to medium are derived.

Let start from the basic expression for the excitation probability of micro target in the field of electromagnetic pulse (EMP):

$$W(\tau) = \frac{c}{(2\pi)^2} \int_0^\infty \sigma(\omega) \frac{|E(\omega,\tau)|^2}{\hbar\omega} d\omega \tag{1}$$

here $\sigma(\omega)$ is excitation cross section of the target, $E(\omega,\tau)$ is Fourier transform of the electric field strength in the pulse, $\tau$ is pulse duration, $c$ is light velocity, $\hbar$ is reduced Planck constant.

One can obtain from (1) the following expression for energy transferred from EMP to micro target as function of pulse duration

$$\Delta \mathrm{E}(\tau) = \frac{c}{(2\pi)^2} \int_0^\infty \sigma(\omega) |E(\omega,\tau)|^2 \, d\omega \, . \tag{2}$$

With the use of optical theorem

$$\sigma(\omega) = \frac{4\pi\omega}{c} \operatorname{Im} \beta(\omega) \tag{3}$$

we arrive instead of (2) at the following expression

$$\Delta \mathrm{E}(\tau) = \frac{1}{\pi} \int_0^\infty \omega \operatorname{Im} \beta(\omega) |E(\omega,\tau)|^2 \, d\omega \, . \tag{4}$$

here $\beta(\omega)$ is dynamic polarizability of the target.

To derive a macroscopic expression for the energy transfer we use the correspondence between polarizability of micro-target and dielectric function of the medium. This correspondence can be traced via two approaches.

The first approach we suggest in the paper [1]. It is based on the following relation

$$\mathrm{Im}\,\beta(\omega) \to \frac{1}{4\pi N_a}\mathrm{Im}\,\varepsilon(\omega) \tag{5}$$

here $N_a$ is concentration of medium atoms, $\varepsilon(\omega)$ is dielectric function (permittivity). After substitution (5) into (4) and integration over all frequencies of monochromatic components of the pulse we obtain the formula for energy transfer from EMP to medium per atom

$$\frac{d}{dN}\Delta\mathrm{E}(\tau) = \frac{1}{4\pi^2 N_a}\int_0^\infty \omega\,\mathrm{Im}\{\varepsilon(\omega)\}\left|E(\omega,\tau)\right|^2 d\omega. \tag{6}$$

This implies the following expression for the transferred energy per unit volume of the medium ($dN = N_a\,dV$)

$$\frac{d\Delta\mathrm{E}(\tau)}{dV} = \frac{1}{4\pi^2}\int_0^\infty \omega\,\mathrm{Im}\{\varepsilon(\omega)\}\left|E(\omega,\tau)\right|^2 d\omega. \tag{7}$$

This formula coincides with the well-known expression describing the dissipation of the energy of a non-monochromatic field in a dispersive medium [10]. We note that in the case under consideration, the non-monochromaticity of the electromagnetic field is due to the finite duration of the electromagnetic pulse.

The second approach is based on the relation

$$\mathrm{Im}\,\beta(\omega) \to \frac{3}{4\pi N_a}\mathrm{Im}\left\{\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}\right\}. \tag{8}$$

Here we use the Lorentz-Lorenz formula for the dynamic polarizability of medium atoms. After substitution (8) in (6) we obtain

$$\frac{d\Delta\mathrm{E}(\tau)}{dV} = \frac{9}{4\pi^2}\int_0^\infty \frac{\omega\,\mathrm{Im}\{\varepsilon(\omega)\}}{\left[\mathrm{Re}\{\varepsilon(\omega)\}+2\right]^2 + \left[\mathrm{Im}\{\varepsilon(\omega)\}\right]^2}\left|E(\omega,\tau)\right|^2 d\omega. \tag{9}$$

Let consider some limiting cases from this expression.

For transparent medium ($\mathrm{Im}\{\varepsilon(\omega)\} \to 0$) we obtain from (9)

$$\frac{d\Delta\mathrm{E}(\tau)}{dV} = \frac{9}{4\pi}\int_0^\infty \omega\,\delta\left(\mathrm{Re}\{\varepsilon(\omega)\}+2\right)\left|E(\omega,\tau)\right|^2 d\omega. \tag{10}$$

After expanding the delta function we get the following equality

$$\frac{d\Delta\mathrm{E}(\tau)}{dV} = \frac{9}{4\pi}\frac{\omega^*\left|E(\omega^*,\tau)\right|^2}{d\,\mathrm{Re}\{\varepsilon(\omega^*)\}/d\omega}. \tag{11}$$

Here frequency $\omega^*$ satisfies the equation $\mathrm{Re}\{\varepsilon(\omega^*)\} = -2$. Note that the same frequency describes the frequency of the surface plasmon on the metal nanosphere surface.

Let consider the following model of permittivity

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \tag{12}$$

here $\omega_p = \sqrt{\frac{4\pi e^2 N_a}{m}}$ is plasma frequency. Formula (12) presumes one excitation of medium with eigenfrequency $\omega_0$. Note that the imaginary part of permittivity (12) is equal to zero in accordance with our above suggestion. In this case the frequency $\omega^*$ is equal to

$$\omega^* = \sqrt{\omega_0^2 + \frac{\omega_p^2}{2+\varepsilon_\infty}} \tag{13}$$

and accordingly

$$\frac{d\Delta \mathrm{E}(\tau)}{dV} = \frac{9}{8\pi}\frac{\omega_p^2}{(2+\varepsilon_\infty)^2}\left|E(\omega^*,\tau)\right|^2. \tag{14}$$

For metal we have $\omega_0 = 0$ and

$$\frac{d\Delta \mathrm{E}(\tau)}{dV} = \frac{9}{8\pi}\frac{\omega_p^2}{(2+\varepsilon_\infty)^2}\left|E\left(\omega = \frac{\omega_p}{\sqrt{2+\varepsilon_\infty}},\tau\right)\right|^2. \tag{15}$$

In the opposite case when $\omega_p << \omega_0$ (the medium consists of two-level systems) we have

$$\frac{d\Delta \mathrm{E}(\tau)}{dV} = \frac{9}{8\pi}\frac{\omega_p^2}{(2+\varepsilon_\infty)^2}\left|E(\omega_0,\tau)\right|^2. \tag{16}$$

For $\varepsilon_\infty = 1$ we arrive at the simple expression

$$\frac{d\Delta \mathrm{E}(\tau)}{dN} = \frac{e^2}{2m}\left|E(\omega_0,\tau)\right|^2. \tag{17}$$

It is important to note that the same as (17) result follows from the start expression (1) if we make the following substitutions:

$$\sigma(\omega) = \frac{2\pi^2 e^2}{mc} f_0\, G(\omega),\ G(\omega) = \delta(\omega-\omega_0),\ f_0 = 1. \tag{18}$$

It is easy to check that result (17) is also obtained when using the first approach. Thus, both approaches in the considered limit give the same results.

Simple analysis shows that in the general case, the use of these approaches leads to significantly different results. The condition for their applicability needs to be studied further.

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