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Toward Diagnostics of the Ion-Acoustic Turbulence in Laser-Produced Plasmas

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ABSTRACT: Electrostatic turbulence frequently occurs in various kinds of laboratory and astrophysical plasmas. Transport phenomena are affected most significantly by a low-frequency electrostatic turbulence – such as, e.g., ionic sound. In this case, for computing profiles of spectral lines, emitted by plasma ions, by any appropriate code for diagnostic purposes, it is necessary to calculate the distribution of the total quasistatic field. For a practically important situation, where the average turbulent field is much greater than the characteristic ion microfield, we develop a robust computational method valid for any appropriate distribution of the ion microfield at a charged point. We show that the correction to the Rayleigh distribution of the turbulent field is controlled by the behavior of the ion microfield distribution *at large fields* - in distinction to the opposite (and therefore, seemingly erroneous) result in the literature. We also obtain a universal analytical expression for the correction to the Rayleigh distribution based on the asymptotic of the ion microfield distribution at large fields at a charged point. Our results can be used for spectroscopic diagnostics of a low-frequency electrostatic turbulence for various kinds of plasmas – especially for laser-produced plasmas.

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1. INTRODUCTION

Electrostatic turbulence frequently occurs in various kinds of laboratory and astrophysical plasmas [1, 2]. It affects transport phenomena in plasmas. It is represented by Oscillatory Electric Fields (OEFs) sometimes called also collective electric fields: they correspond to collective degrees of freedom in plasmas – in distinction to the electron and ion microfields that correspond to individual degrees of freedom of charged particles.

The most significant effect on transport phenomena has a low-frequency electrostatic turbulence. At the absence of a magnetic field, there is only one type of a low-frequency electrostatic turbulence: ion acoustic waves – frequently called *ionic sound*. The corresponding OEF is a broadband field, whose frequency spectrum is below or of the order of the ion plasma frequency

$$\omega_{pi} = (4\pi e^2 N_i Z^2/m_i)^{1/2} = 1.32 \times 10^3\, Z(N_i m_p/m_i)^{1/2}, \quad (1)$$

where N_i is the ion density, Z is the charge state; m_p and m_i are the proton and ion masses, respectively. In the "practical" parts of Eqs. (1, 2 – 4), CGS units are used.

In magnetized plasmas in addition to the ionic sound, propagating along the magnetic field **B**, two other types of low-frequency electrostatic turbulence are possible. One is *electrostatic ion cyclotron wave*, whose wave vector is nearly perpendicular to **B**. Its frequency is close to the ion cyclotron frequency

$$\omega_{ci} = ZeB/(m_i c) = 9.58 \times 10^3\, ZB(m_p/m_i)^{1/2}. \quad (2)$$

Another type is *lower hybrid oscillations* having the wave vector perpendicular to **B**. Its frequency is

$$\omega_{lh} = 1/[(\omega_{ci}\,\omega_{ce})^{-1} + (\omega_{pi})^{-2}]^{1/2}, \quad (3)$$

where ω_{ce} is the electron cyclotron frequency

$$\omega_{ce} = eB/(m_e c) = 1.76 \text{ x}10^7 \text{ B}. \quad (4)$$

From Eq. (3) it is seen that $\omega_{lh} < \omega_{pi}$ always. This means that frequencies of both ionic sound and lower hybrid oscillations are below or of the order of the ion plasma frequency ω_{pi}.

It is usually assumed that hydrogenic radiators perceive OEFs, associated with a low-frequency plasma turbulence as *quasistatic*. Let us discuss this assumption in more detail. The discussion is based on papers [3, 4], the results of which were recently summarized in paper [5].

The physics of the spectral line broadening in plasmas containing OEFs is very rich and complex due to the interplay of a large number of characteristic times and frequencies. There are 7 characteristic frequencies, which can be considered as "elementary" parameters. For our specific discussion here the following 4 frequencies are important.

1. $\Delta\omega$ – detuning from the unperturbed position of a given spectral line of the radiator. It affects the characteristic value of the argument τ of the dipole-dipole correlation function.

2. ω – OEF frequency.

3. γ – homogeneous width of the power spectrum of OEF, which is also the inverse of the OEF *coherence time* τ_F. (It is the width at a fixed wave vector **k** of the OEF – in distinction to the width due to the dependence $\omega(\mathbf{k})$.)

4. $\delta_s(E_0)$ – instantaneous Stark shift at the amplitude value E_0 of OEF (i.e., the shift formally calculated at a static field equal to E_0). For example, $\delta_s(E_0) = a_1E_0$ in the case of the linear Stark effect or $\delta_s(E_0) = a_2E_0^2$ in the case of the quadratic Stark effect; $a_1(j)$, $a_2(j)$ are Stark constants that depend on the set of quantum numbers of the particular states of the radiator. *Here and below the set of quantum numbers is denoted by j.*

On the basis of the above "elementary" frequencies, there occur 2 composite parameters that are characteristic times as follows.

1. $\tau_{QS}(j, E_0, \omega)$ – a characteristic time of the formation of Quasienergy States (QS):

$$\tau_{QS}(j, E_0, \omega) \sim \min(1/(\omega^2\,\delta_s)^{1/3}, 1/\omega). \quad (5)$$

   Being subjected to OEF, the states of the radiator can oscillate with the OEF frequency ω. This effect is described as the emergence of QS, which were introduced in 1967 in papers [6] and [7] (independently of each other). The above formula for $\tau_{QS}(j, E_0, \omega)$ was derived in paper [3]. So, for relatively weak OEF, the QS are formed at the timescale of the order of the period of the OEF $1/\omega$. However, for relatively strong OEF, the QS are formed at a much shorter time scale proportional to $1/E_0^{1/3}$ or to $1/E_0^{2/3}$ in the cases of the linear or quadratic Stark effect, respectively.

2. $\tau_{life}(j, N_e, T_e, N_i, T_i, \gamma, \omega, E_0, \Delta\omega)$ – the lifetime of the exited state of the radiator:

$$\tau_{life}(j, N_e, T_e, N_i, T_i, \gamma, \omega, E_0, \Delta\omega) \sim 1/\Gamma, \quad (6)$$

$$\Gamma = \gamma_e(j, N_e, T_e, \Delta\omega) + \gamma_i(j, N_i, T_i, N_e, \Delta\omega) + \gamma_F(j, \gamma, \omega, E_0), \quad (7)$$

where T_i is the ion temperature, N_e and T_e are the electron density and temperature, respectively. In Eq. (7), Γ is the sum of the *homogeneous* Stark widths due to electrons (γ_e), dynamic part of ions (γ_i), and OEF (γ_F). (The Stark broadening of a spectral line is *homogeneous* when it is the same for all radiators, a typical example being the Stark broadening by the electron microfield; in distinction, the Stark broadening by the quasistatic part F_{qs} of the ion microfield is *inhomogeneous* because different radiators are subjected to generally different values of F_{qs}).

The contribution $\gamma_F^{hf}(j, \gamma, \omega, E_0)$ caused by Langmuir (high-frequency) electrostatic turbulence was calculated in paper [6]. However, the contribution to the homogeneous Stark width from a low-frequency plasma turbulence γ_F^{lf}

is typically much smaller than ($\gamma_e + \gamma_i$), so that this contribution can be neglected. As for the Stark broadening by ions, in the present paper we consider the situation where, from the point of view of a radiating ion, the overwhelming part of perturbing ions is quasistatic. The corresponding well-known criterion can be found, e.g., in review [8] and is presented here in Appendix. This situation is relevant, in particular, to hydrogenlike ions of the nuclear charge Z ~ 10 emitted from laser-produced plasmas of relatively high electron densities $N_e > 10^{20}$ cm^{-3} and temperatures $T_e < 1$ keV.

A criterion for OEF to be considered as quasistatic is the following

$$\tau_{QS}(j, E_0, \omega) >> \min[1/\delta_s(E_0), 1/\Delta\omega, \tau_{life}(j, N_e, T_e, N_i, T_i, \gamma, \omega, E_0, \Delta\omega)]. \quad (8)$$

For hydrogenlike ions of the nuclear charge Z ~ 10 emitted from laser-produced plasmas, the condition (8) is usually fulfilled for OEFs of low-frequency electrostatic turbulence.

One of the primary motivations for the present work is the experiment at JETI Jena laser system described and analyzed in papers [5, 9]. In this experiment two high-powered short-pulse laser beams have been used. The first one created the plasma, and the plasma existed after the laser was off. A delayed high-power short pulse laser was then used to cross the preformed plasma. The study in [5, 9] concerned the effect of the external strong OEF due to this second laser on the Stark profiles of the beta-line of Al XII. The analysis of the experimental Stark profiles was done via advanced simulations: by coupling a lineshape code based on the Floquet-Liouville formalism with a particle-in-cell (PIC) kinetic code (based on the theory of laser-plasma interactions) that provided a spatial distribution of the OEF in the plasma. Laser-plasma interactions significantly enhanced the OEF in the plasma compared to the laser field in vacuum: the high-frequency OEF up to 5 GV/cm in the plasma was found consistent with the experimental line profiles.

The theory of laser-plasma interactions predicts also a possibility of the development of ionic sound (a low-frequency OEF) in this kind of experiments. It was estimated that the electric field of ionic sound could reach values ~ 10 GV/cm [10]. So, it could exceed the characteristic ion microfield by 2 – 3 orders of magnitude. Indeed, e.g., for the conditions of the experiment [5, 9], where N_e was about $3\text{x}10^{20}$ cm^{-3}, the characteristic ion microfield $E_N = 2.6\, Z_p^{1/3} e N_e^{2/3}$ was only about 0.04 GV/cm (here Z_p is the charge of perturbing ions). In order to test this prediction of the theory of laser-plasma interactions, one needs to significantly modify the distribution of the quasistatic field by allowing for a possible dominant contribution of the turbulent field.

In view of the above motivation, we consider below a non-magnetized plasma, so that only one type of the low-frequency electrostatic turbulence is possible: ionic sound. We study the typical situation, where both the turbulent field $\mathbf{E}_t$ and the ion microfield $\mathbf{F}_i$ are quasistatic. We develop a robust method for calculating the distribution of the *total* quasistatic field at a charged point for the case where the average turbulent field is much greater than the characteristic ion microfield. We also reveal a misconception in one of the previous works devoted to this subject.

2. DISTRIBUTION OF THE TOTAL QUASISTATIC ELECTRIC FIELD

In any code designed for calculating spectral line profiles, an important task becomes the averaging over the ensemble distribution W($\mathbf{E}$) of the *total* quasistatic field $\mathbf{E} = \mathbf{E}_t + \mathbf{F}_i$. In other words, the key part of the problem becomes the calculation of W($\mathbf{E}$).

In paper [11] the distribution of a low-frequency turbulent field was derived and shown to be the Rayleigh distribution, which in the isotropic case can be represented in the following form

$$W_t(\alpha, x)dx = 3[6/\pi]^{1/2}\alpha^3 x^2 \exp(-3\alpha^2 x^2/2)]dx. \quad (9)$$

Here

$$x = E_t/E_N \quad (10)$$

is the scaled turbulent field and

$$\alpha = E_N / E_R \quad (11)$$

is the ratio of the "standard" ion microfield E_N to the root-mean-square turbulent field E_R, where

$$E_N = 2\pi(4/15)^{2/3}Z_p^{1/3}eN_e^{2/3} = 3.751\times10^{-7}\, Z_p^{1/3}[N_e(cm^{-3})]^{2/3}\ V/cm. \quad (12)$$

The total quasistatic field **E** results from the vector summation of the two statistically independent contributions: $\mathbf{E} = \mathbf{E}_t + \mathbf{F}_i$. The justification of this has been given in papers [12, 13]. Therefore a *general* distribution $W_g(\mathbf{E}/E_N)$ of the total field is a convolution of the distribution $W_t(\mathbf{E}_t/E_N)$ of the turbulent field with the distribution $W_i(\mathbf{F}_i/E_N)$ of the ion microfield (subscript "g" in $W_g(\mathbf{E}/E_N)$ stands for "general"):

$$W_g(\beta)d\beta = [\iint d\mathbf{x}d\mathbf{u}\ W_i(\mathbf{u})W_t(\mathbf{x})\delta(\beta - \beta_s)]d\beta, \quad \beta = \mathbf{E}/E_N,\ \mathbf{x} = \mathbf{E}_t/E_N\ ,\ \mathbf{u} = \mathbf{F}_i/E_N\ . \quad (13)$$

Here

$$\beta_s = |\mathbf{x} + \mathbf{u}| = (x^2 + u^2 - 2ux\cos\theta)^{1/2}, \quad (14)$$

where θ is the angle between vectors **u** and **x**.

Equation (13) represents a general result valid for both isotropic and anisotropic distributions of the turbulent field. A general form of anisotropic, but axially-symmetric distribution $W_t(\mathbf{x})$ of the turbulent field was derived in paper [14] (and was later called in the literature Sholin-Oks' distribution). Calculations of $W_g(\beta)$ using Sholin-Oks' axially-symmetric distribution of $W_t(\mathbf{x})$ will be published elsewhere. In the present paper we consider the case of the isotropic distribution of the turbulent field, so that it is represented by Eq. (9). In this case, the distribution of the total field $W(\beta)$ will be also isotropic (since both $W_i(\mathbf{u})$ and $W_t(\mathbf{x})$ are isotropic) and can be written as

$$W(\beta)d\beta = [\iint d\mathbf{x}d\mathbf{u}\ W_i(\mathbf{u})W_t(\mathbf{x})\delta(\beta - \beta_s)]d\beta. \quad (15)$$

In paper [11] the study was focused at so-called ideal (or "non-coupled") plasmas where the kinetic energy is by several orders of magnitude greater than the potential energy. It is manifested by a very large number of perturbing ions N_{iD} in a sphere of the electron Debye radius r_{De}:

$$N_{iD} = (4\pi/3)N_e r_{De}^3/Z_p >> 1, \quad (16)$$

where

$$r_{De} = [T_e/(4\pi e^2 N_e)]^{1/2}. \quad (17)$$

A practical formula for the quantity N_{iD} is*/:

$$N_{iD} = 1.7181\times10^9\ [T_e(eV)]^{3/2} / \{Z_p[N_e(cm^{-3})]^{1/2}\}. \quad (18)$$

The study in paper [11] was designed for application to experiments, where hydrogen line profiles were emitted from turbulent plasmas of electron densities ~ $10^{14} – 10^{15}$ cm^{-3} and temperatures of several eV. For example, for $N_e = 3\times10^{14}$ cm^{-3}, $T_e = 4 – 5$ eV, and $Z_p = 1$, Eq. (18) yields $N_{iD} \sim 10^3$. For this kind of plasmas, where N_{iD} was "dramatically" greater than unity, the ion microfield distribution was chosen in paper [11] as the Holtsmark distribution [15]:

$$W_H(u) = (2u/\pi)\int_0^\infty dx\ \sin ux\ \exp(-u^{3/2}). \quad (19)$$

The Holtsmark distribution describes a transition from the Gaussian distribution for weak fields u << 1 to the binary distribution (nearest-neighbor distribution) of strong fields u >> 1, as noted in review [8]. Indeed, the weak-field part of the Holtsmark distribution $W_H \approx 4u^2/(3\pi)$ is due to a cumulative effect of large number of perturbers – therefore, like any sum of a large number of random quantities, it follows the Gaussian distribution (i.e., its starting part ~ u^2). In the opposite limit of u >> 1, where $W_H = 15/[4(2\pi)^{1/2}u^{5/2}] = 1.496/u^{5/2}$, only the nearest neighbor controls the distribution.

For the distribution of the total quasistatic field, defined by Eq. (15), the following result was obtained in paper [11] for ideal plasmas – by performing analytically several integrations in (15):

* / N_{iD} is related to another coupling parameter Γ used in plasma physics as follows: $N_{iD} = [Z_p/(3\Gamma)]^{3/2}$.

$$W(\alpha,\beta) = [3/(2\pi)^{1/2}]\alpha\beta\int_0^\infty du\{\exp[-3\alpha^2(\beta-u)^2] - \exp[-3\alpha^2(\beta+u)^2]\}W_H(u)/u. \quad (20)$$

In the present paper we focus at weakly non-ideal ("weakly-coupled") plasmas, where the quantity N_{iD} is still greater than unity, but not dramatically greater: $N_{iD} \sim 10$. A particular application could be, e.g, the laser-produced plasma experiment described in papers [5, 9], characterized by $N_e \sim 3 \times 10^{20}$ cm^{-3}, T ~ 150 eV, the majority of radiating and perturbing ions having the charge Z = 12, so that $N_{iD} = 15$. In this situation, first of all, the ion microfield distribution differs significantly from the Holtsmark distribution. The second distinction is that the ion microfield distribution should be calculated at a point of the charge Z ~ 10, and in plasmas of $N_{iD} \sim 10$ this distribution would be noticeably different from the distribution at a neutral point because of ion-ion correlations.

Therefore, our starting point is the following expression for the distribution W(α, β) of the total quasistatic field, which is similar to Eq. (20), but without assuming any specific form of the ion microfield distribution $W_i(u)$:

$$W(\alpha,\beta) = [3/(2\pi)^{1/2}]\alpha\beta\int_0^\infty du\{\exp[-3\alpha^2(\beta-u)^2] - \exp[-3\alpha^2(\beta+u)^2]\}W_i(u)/u. \quad (21)$$

In other words, $W_i(u)$ could be any ion microfield distribution at a charged point, calculated by any appropriate code (e.g., by using the APEX method [16]).

Equation (21) can be rewritten in the form:

$$W(\alpha, \beta) = [3/(2\pi)^{1/2}]\alpha\beta\, \exp(-3\alpha^2\beta^2)\, x \quad (22)$$

$$\int_0^\infty du\{\exp[-3\alpha^2(u^2 - 2\beta u)/2] - \exp[-3\alpha^2(u^2 + 2\beta u)/2]\}W_i(u)/u.$$

For the situation considered in the present paper, where the average turbulent field is much greater than the characteristic ion microfield ($\alpha = E_N/E_R << 1$), it is appropriate to expand both exponentials in the integrand in (22) in Taylor series. Keeping terms up to (including) those ~ u^4, we obtain

$$W(\alpha, \beta) = W_t(\alpha, \beta)\, [1 - (3\alpha^2/2)(1 - \alpha^2\beta^2)M_{i2}], \quad (23)$$

where $W_t(\alpha, \beta)$ is the Rayleigh distribution given by Eq. (9) and M_{i2} is the second moment of the ion microfield distribution:

$$M_{i2} = \int_0^\infty du\, u^2 W_i(u). \quad (24)$$

Equation (23) shows that at $\alpha = E_N/E_R << 1$, in the first approximation the distribution of the total quasistatic field reduces to the Rayleigh distribution, as should be expected. More important is that Eq. (23) also shows that the first nonvanishing correction to the Rayleigh distribution (the second term in brackets in (23)) is controlled by the second moment of the ion microfield distribution.

Two important comments should be made at this point. First, Eq. (23) has a *great computational advantage* compared to Eq. (21). Indeed, while employing Eq. (21), one would have to choose at least ~ 10^2 values of â and at least ~ 10 values of α. So, one would have to use $W_i(u)$, calculated numerically by some code, at least ~ 10^3 times. In distinction, while utilizing Eq. (23), one would have to use $W_i(u)$ *only once* – for calculating the second moment of $W_i(u)$. Thus, the employment of Eq. (23) instead of Eq. (21) makes *much more robust* any code for calculating spectral line profiles in turbulent plasmas.

The second comment, following from Eq. (23), is related to paper [17]. This paper was also devoted to calculations of the distribution of the total quasistatic field for the situation where the average turbulent field is much greater than the characteristic ion microfield. The authors of [17] suggested an approximate method where the correction to the Rayleigh distribution was controlled by the behavior of the ion microfield distribution $W_i(u)$ *at small fields* ($u<<1$). However, from Eqs. (23), (24) it can be seen that this was a misconception. Indeed, the integral in (24) accumulates most of its value at $u >> 1$: it converges only because at very large values of u, the ion-ion correlations – the repulsion between the radiating and perturbing ions – "kill" the integral. Thus, in reality the correction to the Rayleigh distribution is controlled by the behavior of the ion microfield distribution $W_i(u)$ *at large fields* rather than at small fields.

In view of the above second comment, we can use the well-known asymptotic of $W_i(u)$ at $u >> 1$ at a charged point and obtain a universal analytical result for the second moment of $W_i(u)$ and thus for the correction to the Rayleigh distribution. This is presented in the next section.

3. ASYMPTOTIC UNIVERSAL ANALYTICAL RESULT

Analytical results for the large-field asymptotic of the ion microfield distribution at a charged point were presented in papers [18, 19]*/. We consider here plasmas where the charge of radiating ions Z_r is the same as for perturbing ions: $Z_r = Z_p \equiv Z$. For this case, the corresponding formula, obtained in [18, 19] with the allowance for ion-ion correlations and for the screening of the ion field by plasma electrons, has the form:

$$W_{i,as}(u) = (q/Z^2)(u/Z + v^2/2)^{-5/2} \text{ x} \tag{25}$$

$$\exp\{-[T_e/(2qT_i)]\, Z^2v^2(u/Z + v^2/2)^{1/2} \exp[-(1 + ZT_e/T_i)^{1/2}/(u/Z + v^2/2)^{1/2}]\}.$$

Here

$$q = 15/[4(2\pi)^{1/2}] = 1.496, \quad v = r_0/r_{De}, \quad r_0 = [15/(4N_e)]^{1/3}/(2\pi)^{1/2}. \tag{26}$$

(the quantity r_0 is defined such that it is close to the mean interionic distance).

The quantity v is yet another indicator of the proximity of a plasma to the non-ideality (i.e., the coupling indicator). It is related to the number of perturbing ions N_{iD} in a sphere of the electron Debye radius as follows:

$$N_{iD} = 0.9974/(Zv^3). \tag{27}$$

It is seen that $v << 1/Z^{1/3}$ corresponds to ideal plasmas ($N_{iD} >> 1$), while $v > 1/Z^{1/3}$ corresponds to strongly-coupled plasmas. A practical formula for the quantity v has the form:

$$v = 8.98\text{x}10^{-2}\, [N_e(\text{cm}^{-3})]^{1/6} / [T_e(\text{K})]^{1/2}. \tag{28}$$

If the following condition is met

$$u >> Zv^2/2, \tag{29}$$

the asymptotic formula (25) simplifies to

$$W_{i,as}(u) = (qZ^{1/2}/u^{5/2}) \exp(-ku^{1/2}), \tag{30}$$

where

$$k = T_eZ^{3/2}v^2/(2qT_i). \tag{31}$$

We emphasize that the condition (29) is usually much less restrictive than the validity condition ($u >> 1$) of the formula (25). For example, for the laser-produced plasma experiment described in papers [5, 9], characterized by $N_e \sim 3\text{x}10^{20}$ cm^{-3}, T ~ 150 eV, Z = 12, the inequality (29) yields $u >> 0.2$. Thus, for a broad range of weakly coupled plasmas, the simplified asymptotic from Eq. (30) can be used with a very good accuracy instead of the asymptotic from Eq. (25).

Using this asymptotic, the second moment of the ion microfield distribution can be approximately represented in the form:

*/ The current status of the ion microfield distribution studies can be found in review [20].

$$M_{i2} = I_1 + I_2, \quad I_1 = \int_0^C du\ u^2 W_i(u); \quad I_2 = \int_C^\infty du(qZ^{1/2}/u^{1/2})\exp(-ku^{1/2}), \tag{32}$$

where C ~ 1. The second integral I_2 can be calculated analytically yielding:

$$I_2 = (2qZ^{1/2}/k)\exp(-C^{1/2}k). \tag{33}$$

The central point of this section is that for a broad range of weakly coupled plasmas, the integral $I_2 \sim (2qZ^{1/2}/k) >>1$, while the first integral in (32) $I_1 \sim 1$. Therefore, the second moment of the ion microfield distribution, calculated by any appropriate code, can be well approximated by the analytical result (33) as long as k < 1 (so that I_2 from (33) is not sensitive to a particular choice of C ~ 1) and

$$2qZ^{1/2}/k = 4q^2T_i / (Zv^2T_e) >> 1. \tag{34}$$

Under this condition, which is satisfied for a broad range of weakly coupled plasmas, the second moment of the ion microfield distribution can be accurately calculated by the analytical formula (33) regardless of the particular behavior of this distribution at small fields. In this sense, formula (33) is *universal*.

As an illustration of the accuracy of the analytical result (33), we compare it with the exact calculation of the second moment for the nearest neighbor (binary) distribution $W_{iB}(u)$ at a charged point*/. The latter distribution in the normalized form is given by the following expression (under the condition (29)):

$$W_{iB}(u) = u^{-5/2}\exp(-u^{-3/2} - ku^{1/2}) / \int_0^\infty du[u^{-5/2}\exp(-u^{-3/2} - ku^{1/2})]. \tag{35}$$

Figure 1 shows two calculated dependences of the second moment of the above binary distribution versus parameter k. Solid line represents the result obtained using the large-field asymptotic with the choice of the lower limit C = 1, dashed line – the exact result. It is seen that the asymptotic method is very accurate as long as k < 1.

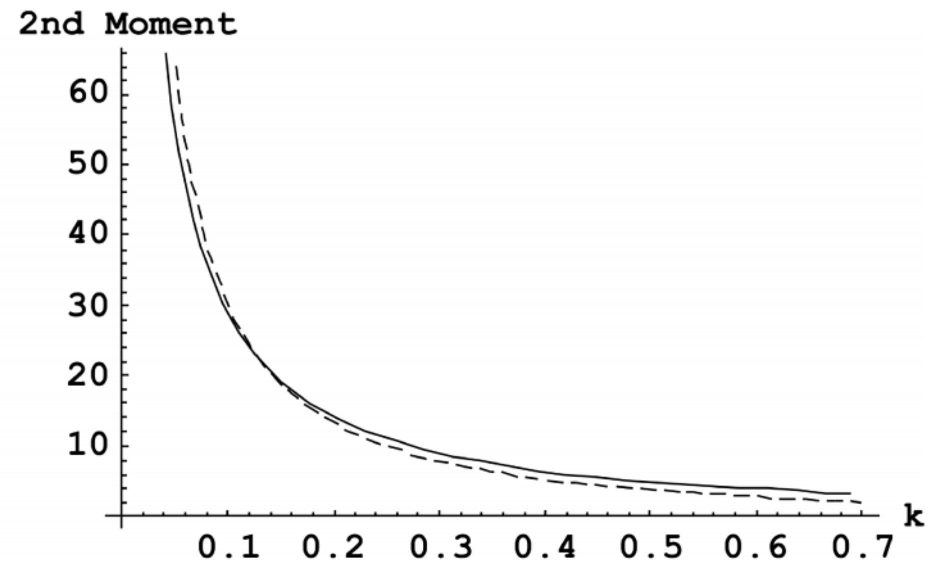

Figure 1: The second moment of the nearest neighbor distribution at a charged point versus parameter $k = T_eZ^{3/2}v^2/(2qT_i)$, representing the degree of the non-ideality of plasmas: solid line – the result obtained using the large-field asymptotic of the nearest neighbor distribution with the choice of the lower limit C = 1; dashed line – the exact result. Here Z is the charge of plasma ions, parameters v and q are defined in Eq. (26)

*/ This distribution has two important common features with ion microfield distributions calculated by any appropriate code for weakly coupled plasmas: the asymptotic at the large fields and a significant shift of the maximum toward small fields (compared to the Holtsmark distribution).

Figure 2 shows dependences of the second moment of the above binary distribution, calculated by using the asymptotic formula (33), versus parameter k – for two different choices of the lower limit of the integration: C = 0.5 (solid line) and C = 2 (dashed line). It is seen that the results are practically insensitive to a particular choice of the lower limit of the integration c within the requirement C ~ 1 – as long as k < 1.

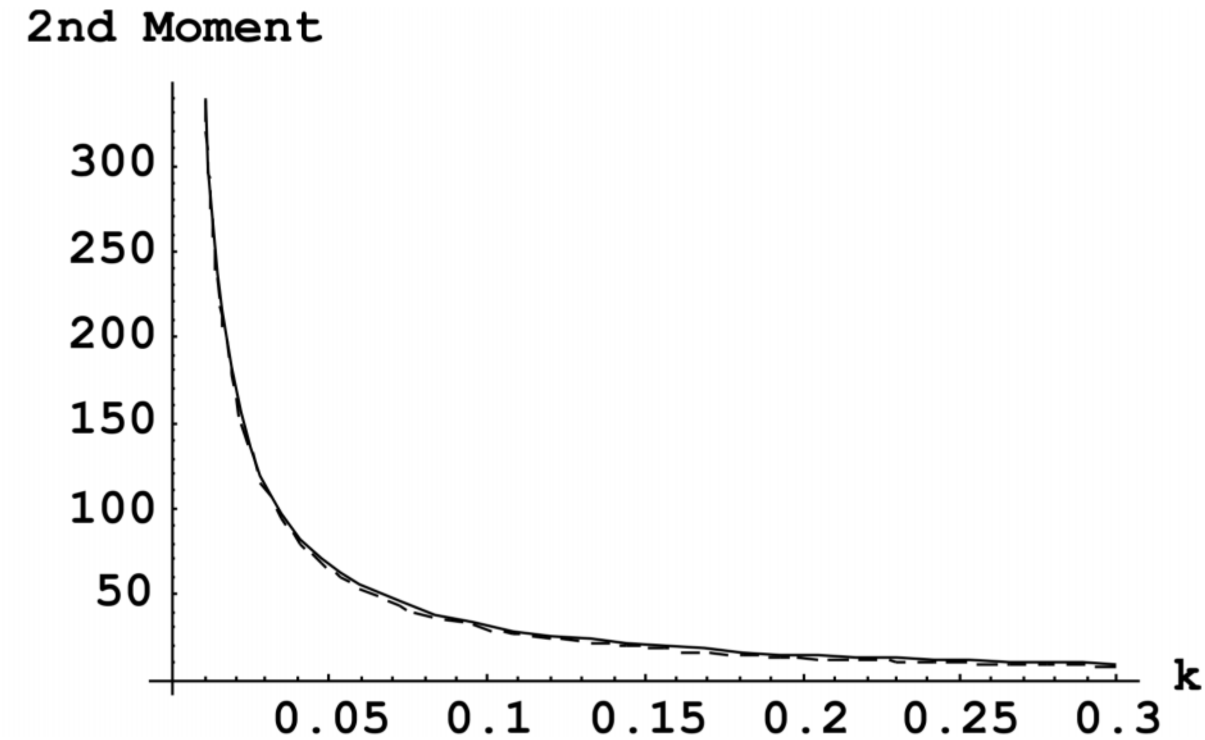

Figure 2: The second moment of the nearest neighbor distribution at a charged point, calculated using the large-field asymptotic of this distribution, versus parameter $k = T_e Z^{3/2} v^2/(2qT_i)$, representing the degree of the non-ideality of plasmas: solid line – with the choice of the lower limit C = 0.5; dashed line – with the choice of the lower limit C = 2. Here Z is the charge of plasma ions, parameters v and q are defined in Eq. (26)

Finally we emphasize that it is appropriate to calculate the second moment of the ion microfield distribution using the approximate analytical result (33) because the second moment enters only the correction term in Eq. (23) for the distribution of the total quasistatic field. Any correction to the analytical result (33) would enter Eq. (23) only as “a correction to the correction”.

4. CONCLUSIONS

We considered plasmas containing a low-frequency electrostatic turbulence (such as, e.g., ionic sound), where spectral lines are emitted by ions – e.g., by multicharged ions in laser-produced plasmas. For computing spectral line profiles for this kind of plasmas by any appropriate code for diagnostic purposes, it is necessary to calculate the distribution of the total quasistatic field. For a practically important situation, where the average turbulent field is much greater than the characteristic ion microfield, we developed a robust computational method valid for any appropriate distribution of the ion microfield at a charged point.

We also showed that the correction to the Rayleigh distribution of the turbulent field is controlled by the behavior of the ion microfield distribution *at large fields* . In this way we demonstrated that the authors of paper [17] seem to make a conceptual error by suggesting an approximate method where the correction to the Rayleigh distribution was controlled by the behavior of the ion microfield distribution at small fields.

Finally, we obtained a universal analytical result for the correction to the Rayleigh distribution based on the asymptotic of the ion microfield distribution at large fields. In the present paper we used the asymptotic at a charged point for plasmas where all ions have the same charge. The corresponding results for plasmas having ions of two different charges will be published elsewhere.

We believe that our work created a basis for spectroscopic diagnostics of a low-frequency electrostatic turbulence for various kinds of plasmas – especially for laser-produced plasmas, such as, e.g., in experiments [5, 9].

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Appendix. Validity Criterion for the Quasistatic Description of the Ion Microfield

In accordance to the review [8], the quasistatic description of the ion microfield is valid if at least one of the following two inequalities are satisfied:

$$V_{Ti}(N_e/Z_p)^{1/3} << \max[Z_p^{1/3} n^2 \hbar N_e^{2/3}/(Z_r m_e),\ \gamma_e,\ \omega_{pe}], \tag{A.1}$$

$$Z_r m_e V_{Ti}^2/(Z_p n^2 \hbar) << \Delta\omega. \tag{A.2}$$

Here V_{Ti} is the mean thermal velocity of the relative motion in the pair "perturbing ion – radiating atom/ion", n is the principal quantum number of the level, from which a spectral line originates, γ_e is the dynamical Stark width due to plasma electrons, $\Delta\omega$ is the detuning from the line center, and ω_{pe} is the plasma electron frequency:

$$\omega_{pe} = (4\pi e^2 N_e/m_e)^{1/2} = 5.6414\text{x}10^4\ [N_e(\text{cm}^{-3})]^{1/2}\ . \tag{A.3}$$

Equations (A.1) and (A.2) contain four validity conditions for the quasistatic description of the ion microfield – *the fulfilment of any of the four conditions is sufficient*. The physical meaning of these conditions is the following.

The quantity in the left side of Eq. (A.1) is the characteristic frequency of variation of the ion microfield in its multi-particle description. The first quantity (out of three) in the right side of (A.1) is the instanteneous Stark shift at the typical strength of the ion microfield; its comparison with the left side of (A.1) is called a "modulation-type" condition for the quasistatic approximation. The comparison of the left side of (A.1) with the second quantity (γ_e) is called a "damping-type" condition for the quasistatic approximation. The third quantity (ω_{pe}) in the right side of (A.1) manifests the screening of the ion microfield by electrons.

The quantity in the left side of Eq. (A.2) is a so-called ion Weisskopf frequency. The Weisskopf frequency is a concept related to the binary (rather than multi-particle) description of the ion microfield. In the binary description, which is appropriate for sufficiently far wings of a spectral line, the characteristic frequency of variation of the ion microfield is the Weisskopf frequency. If it is smaller than the detuning $\Delta\omega$ from the unperturbed position of the spectral line, this would be yet another sufficient condition for the quasistatic approximation.

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