COMP 5660/6660 - Evolutionary Computing - Lecture Slides

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- Step 2: solve computationally in abstract model
- "Everything Should Be Made as Simple as Possible, But Not Simpler"[1]
- Step 3: map solution back to real-world

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- Mad Engineer Thought Experiment

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- Unimodal versus multimodal problems

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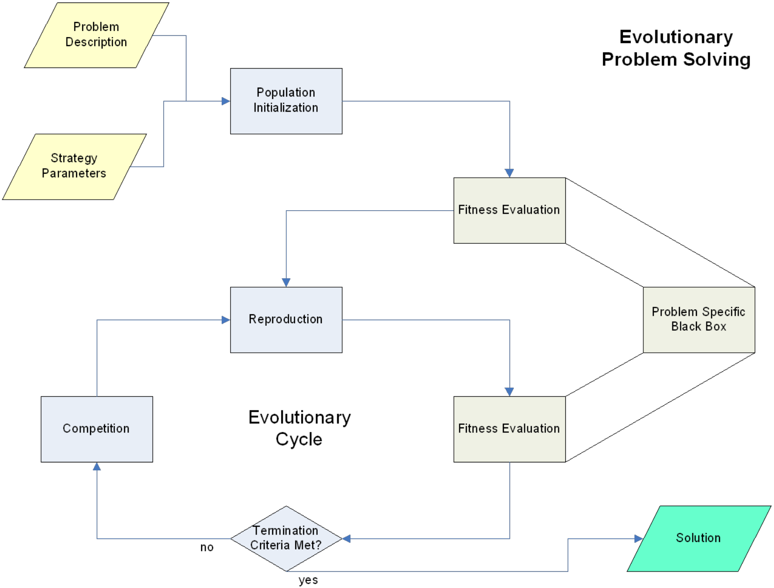
- **Evolutionary Algorithms (EAs)** can be described as a class of *stochastic*, *population-based* BBSAs inspired by *Evolution Theory*, *Genetics*, and *Population Dynamics*

Evolutionary Cycle



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- Solution availability during computation
- Robustness
- Inherent parallelism

EA Cons

- Fitness function and genetic operators often not obvious
- Premature convergence
- Computationally intensive
- Difficult parameter optimization

Biological Metaphors - Darwinian Evolution

- Macroscopic view of evolution
- Natural selection
- Survival of the fittest
- Random variation
- Genetic drift

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- Alleles - variant forms of a gene

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- F does not need to be injective, but realize there is less to search if F is injective so there should be sufficient compensation, such as limiting F(G) to valid solutions in a constraint satisfaction problem.

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2. Modify $fitness(p)$ to exclude items that would exceed C_{max}

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