Diffraction of Electrons

Object: Verify that electrons are waves; i.e., that they diffract just like light waves. This lab is then used to measure their wavelength or, alternatively, measure the spacing between the layers of carbon atoms in polycrystalline graphite that causes the diffraction.

Apparatus: Electron diffraction tube (electron gun, thin graphite screen, fluorescent screen), power supply for filament and accelerating voltage, wires, ammeter, and flexible plastic ruler with scale in mm.

Introduction

It is well known that X-rays, being light waves, diffract from crystalline surfaces just like visible light reflecting from a diffraction grating. The condition for constructive interference is given by Bragg's Law:

$$2d\sin\theta = m\lambda; \quad m = 1,2,3... \tag{1}$$

where θ is the so-called **grazing angle of incidence** (see Fig. 27.13 on p. 877 of your book; it's not the same as the usual angle of incidence, but its **complement**) and *d* is the distance between the planes of atoms in the crystal. X-ray diffraction is commonly used to determine the molecular structure of molecules, and perhaps is most famous for identifying the helical structure of the DNA molecule, the foundation of molecular genetics.

Remarkably, electrons (which we know are particles of charge -1.6×10^{-19} C and mass 9.1×10^{-31} kg) **also** show the identical property of diffraction when a beam of them shines on a crystal. Therefore we conclude they also have a wavelength. About 100 years ago, Louis de Broglie conjectured that the wavelength of a particle is given by

$$\lambda = \frac{h}{mv}$$

where *h* is Planck's constant, *m* is the mass of the particle, and *v* its velocity.

How do we find this wavelength? While we don't measure the velocity of the particle directly, we DO know its kinetic energy if we accelerate the particle through a potential difference ΔV:

$$KE = \frac{1}{2}mv^2 = e\Delta V \Rightarrow mv = \sqrt{2em\Delta V} \tag{2}$$

giving

$$\lambda = \frac{h}{\sqrt{2em\Delta V}} \tag{3}$$

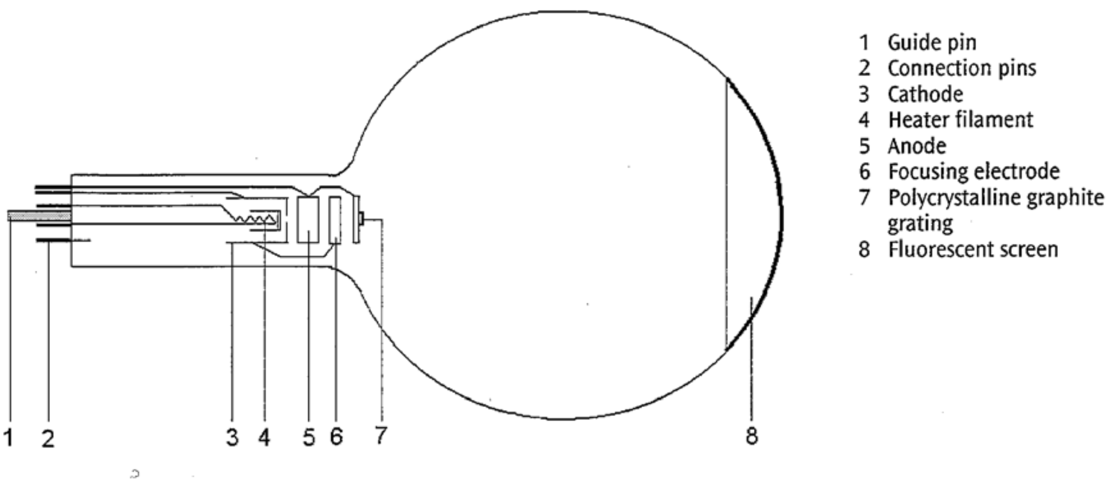
To actually measure the wavelength using diffraction and interference, we use Bragg's law (Eq. 1) where we make use of only first-order diffraction (m=1) such that

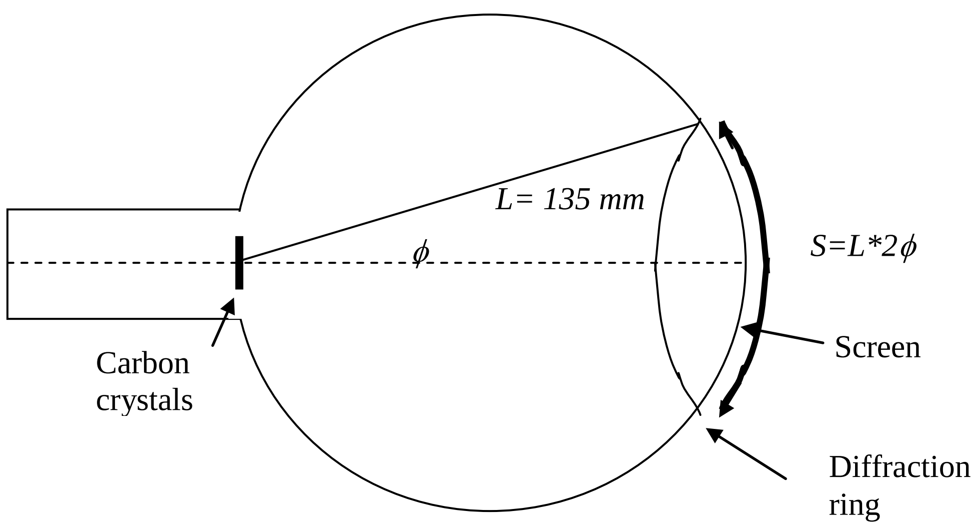
$$\sin\theta = \frac{\lambda}{2d} = \frac{h}{2d\sqrt{2em\Delta V}}, \tag{4}$$

we can see that by measuring ΔV and θ, we can determine the spacing d between the atomic layers in the crystal illuminated by the electron beam. We will check this against the wavelength from de Broglie's conjecture (Eq. 3).

Experiment

Shown below is a sketch of the electron diffraction tube. Current is supplied to the filament (4) to heat it up so that it emits electrons. The electrons gain kinetic energy from a voltage difference ΔV applied between the cathode (3) and anode (5). The electrons strike a thin screen of graphite (7). Most go straight through and make a bright spot at the center of the fluorescent screen on the other side of the tube (8).





Some of the electrons strike the graphite crystals at the correct angle θ for constructive interference (see Eq. (1)), and change their direction by an **angle $\phi = 2\theta$** (Look at Fig. 27.13 to convince yourself that the diffracting electrons change their direction by **twice** the grazing angle). The electrons hit the screen some distance away from the central spot, as shown in the figure below.

- Turn the knob on the power supply all the way counterclockwise (down)

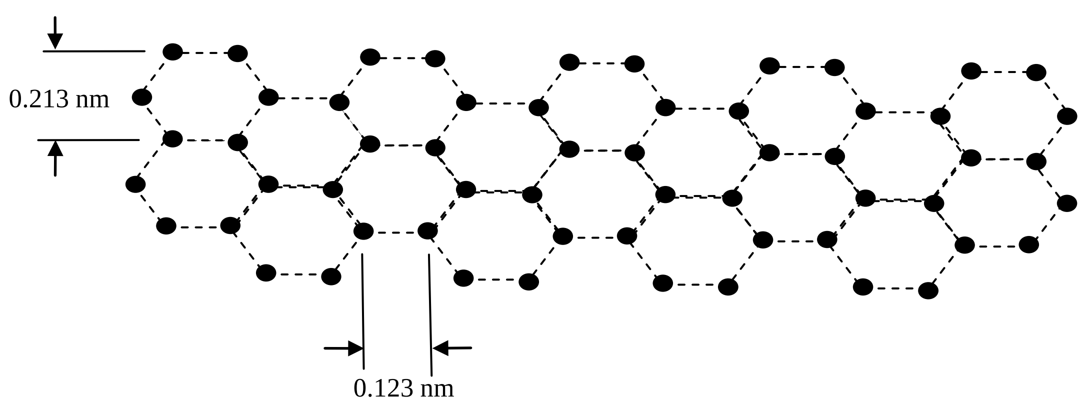
- Push on-off button on power supply to turn it on, and wait about 1 minute for filament to warm up. If the room is not too bright, you can see the yellow glow of the filament.

- Turn the high voltage control knob up slowly, and soon you will see a small spot in the center of the screen. This represents those electrons that go straight through, undiffracted.

As you continue to increase the voltage (you only need to go to 4.0 kV, indicated on the meter on the power supply), you will see 2 faint circular rings around the central spot. These rings represent the diffracted beams.

1. So, if the electrons are supposed to diffract at a specific angle from the crystal, why do they make a **circular** pattern on the screen, and not a single spot at angle ϕ from the central point? The graphite (crystalline carbon) target is composed of many crystals oriented at random directions with respect to one another. It is difficult and probably expensive to make large single crystals (which we would pass on to you in your lab fee) of graphite. So this lab uses a powder of micro-crystals formed into small target. Electrons those electrons that strike a crystal oriented at the Bragg angle (see Eq. 1) with respect to the incident beam will diffract; otherwise they go straight through. Since the millions of crystals are oriented randomly with respect to the electron beam, the diffracting electrons can lie on a cone of angle ϕ, rather than a single line.

2. Why two rings? Is it the first and second order diffraction? Actually not. The hexagonal arrangement of carbon atoms in the graphite crystal creates two diffraction planes of different spacing d: d_0 = 0.213 nm and d_1 = 0.123 nm.



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Diffraction of Electrons
Data Sheet

Starting at a value of ΔV = 4.0 kV, measure (in mm or cm) the arc length *S* of the **diameter** of the **two** diffraction rings with your flexible ruler. Wrap the ruler along the surface of the glass, making sure it goes through the central spot so you can be sure you are measuring along a diameter. The diffraction angle (**in radians, not degrees; recall that 2π radians = 360°**) is *approximately* $\phi = S/(2L)$, where *L* is given to us by the tube manufacturer to be 135 mm.

Measure the two diameters for 4 different voltage values (try ΔV = 4.0 kV, 3.6 kV, 3.2 kV, and 2.8 kV). The rings will get dimmer as you lower ΔV, and you may have to turn out the lights, or put your head and the screen under a thick cloth to block out the ambient light. Record your values in the table below, and perform the necessary calculations to obtain the wavelengths from diffraction and from de Broglie's formula.

ΔV(kV)	S_{inner}	S_{outer}	ϕ_{inner}	ϕ_{outer}
4.0				
3.6				
3.2				
2.8				

It has been stated that an inner and outer ring are formed due to two diffraction planes of different spacing *d*: d_0 = 0.213 nm and d_1 = 0.123 nm. But which spacing is responsible for each of the two rings? Provide your reasoning for which spacing is responsible for each ring. [Hint: see equation (4)]

Ring	Spacing responsible for creating ring (d_0 = 0.213 nm or d_1 = 0.123 nm.
Inner	
Outer	

ΔV(kV)	λ_{inner}(nm)	λ_{outer}(nm)	$\lambda_{de\ Broglie}$(nm)
4.0			
3.6			
3.2			
2.8			

On a sheet of graph paper or using Excel, make an x-y plot of the three wavelength measurements (y-axis) vs $(\Delta V)^{-1/2}$. Use different symbols to plot the three different ways you determined λ. Fit a line to the points corresponding to $\lambda_{de\ Broglie}$.

On a sheet of graph paper or using Excel, make an x-y plot of $\lambda_{de\ Broglie}$ vs $\sin\theta$. Use different symbols to plot data that corresponds to the inner and outer rings. Fit a line to each of the data sets, and determine the slope of the line.

Analysis and discussion

1. Changing the anode voltage causes the diffraction rings to change in diameter. Does reducing the voltage make the rings larger or smaller? How does this support the idea that the electron's wavelength increases as the momentum is reduced?

2. Draw some conclusions from your λ vs $(\Delta V)^{-1/2}$plots. Do your calculated values for λ_{inner} and λ_{outer} reasonably match the theoretical $\lambda_{de\ Broglie}$ at each voltage level? Explain why we used a linear fit. What does the slope represent? Using your data, calculate a percent error for your calculated values of λ_{inner} and λ_{outer} at each voltage level. Use the theoretical $\lambda_{de\ Broglie}$ as your standard value at each voltage level.

3. What physical constants of the apparatus can be extracted from the slopes of your $\lambda_{de\ Broglie}$ vs $\sin\theta$ plots? [Hint: See equation (1).] Use the slopes to calculate the physical constants, and compare your experimental findings to the actual values that you have been provided. Also, calculate a percent error.

4. Comment on what you think could be sources of error (or lack of precision) that could lead to the differences you observe in question 2 above. Try to think of at least three.