I R A M P
3(1), June 2012, pp. 37-50

ISSN: 2229-3159

The Enhancement of Orbital Electron Captures by Nuclei in Plasmas

M. YU. ROMANOVSKY

A. M. Prokhorov General Physics Institute of RAS, Vavilov str., 38, 111991 Moscow, Russia
E-mail: slon@kapella.gpi.ru; myrom@gpad.ac.ru

ABSTRACT: After abrief review of widely investigated effect of beta-decay enhancementby external electric (laser) field, we devote most attention to a newly proposed effect of the capture of atomic orbital electrons by nuclei in the external electric field. Electric fields are sorted by theirorigin: laser field, static field, plasma microfield, the most interesting results are connected with the last one. Since allowed captures of *s*-state bound electrons from inner shells can only be weakened by the external electric field,the probability of various forbidden unique electron captures involving orbital electron non-*s*-states is examined in the simple Slater approximation. The calculations indicate that electric fields with the amplitude larger than the atomic value cansignificantly accelerate the first forbidden capture of *p* electrons, whereas electric fields with the amplitude (much) smaller than the atomic value can significantly accelerate the second forbidden capture of *d* electrons. Special attention is devoted to consideration of plasma effects on the orbital electrons capture rate including the effects on probable double electrons neutrinoless capture by nuclei. It is shown that the enhancement factor of double neutrinoless electron capture may be up to the 10^7 in theproperly ionized plasma.

1. INTRODUCTION. ENHANCEMENT OF BETA DECAY BY EXTERNAL ELECTRIC FIELD

The enhancement of beta decay of particles (a process inverse to an orbital electron capture by nuclei) induced by an external electric field has been studied for more than four decades since the pioneering works [1, 2]. The enhancement of nuclear betadecay was explained by the transformation of the wave function of a free emitted electron in the (high) electromagnetic field [3-5]. Indeed, the transformed wave-function of the final electron states can be written as

$$\Psi_k(r,t) = \exp\left[i\boldsymbol{kr} - \frac{i}{2m\hbar}\int\left(\hbar\boldsymbol{k} - \frac{eA(t')}{c}\right)dt'\right] \tag{1}$$

where Ψ_k is this wave-function, $\boldsymbol{r}$ is the radius-vector, t is time, $\boldsymbol{k}$ is the wave-vector, $\hbar$ is Plank constant, m is electron mass, c is the speed of light, $\boldsymbol{A}$ is the vector-potential of an external (electromagnetic) field. The term "final state transformation" means that only particles in the final state strongly couple to the external field, whereas the initial state is not influenced. The strength of any external field is much weaker than the Coulomb field on the nuclear surface so that theexternal field essentially affects the electronic wave-function only,but not the nuclear wave-function in the beta decay. Since the electron is generated by weak interaction, the external field modifies the final state wave-function of the emitted electrononly.

For any decay, the value of electric field Enecessary to produce a dramatic enhancement of beta decay rates is about Schwinger field E_{Sch}

$$E_{Sch} = \frac{m^2c^3}{e\hbar} \cong 1.3\cdot 10^{16} V/cm \tag{2}$$

Such electric field is unreachable in laboratory now, therefore the perturbation theory over the parameter E/E_{Sch} is valid. In this approach, the first perturbed term for the allowed beta decay rate W_{0E} is [6]

$$W_{0E} \cong W_0 \left[1 + \frac{35}{64} \left(\frac{E}{E_{Sch}} \right)^2 \left(\frac{mc^2}{\varepsilon_0} \right)^3 \right] \tag{3}$$

where ε_0 is the maximal energy of the expelled beta-electron, W_0 is unperturbed rate of the corresponding allowed beta decay. For $_3$T (ε_0 = 18,6 keV) and maximally achieved now amplitude of electric field (in femtoseconds superstrong laser pulses) $E \sim 10^{13}$ V/cm the enhancement (the second term in square brackets in the right part of (3)) is about $2 \cdot 10^{-3}$.

For the first forbidden unique beta decay, the enhancement factor looks like in Eq. (3). If W_1 is the rate of this decay in the absence of external field is [7, 8]

$$W_{1E} \cong W_1 \left[1 + \left(\frac{231}{64} \right) \frac{15mc^2 + 8\varepsilon_0}{11mc^2 + 8\varepsilon_0} \left(\frac{E}{E_{Sch}} \right)^2 \left(\frac{mc^2}{\varepsilon_0} \right)^3 \right] \tag{4}$$

The same form has the enhancement for the second forbidden decay, etc.[1]

The physical reason why electric field of about Schwinger field causes a dramatic increase of the beta decay rates is clear. Indeed, Schwinger field produces electron-positron pairs from vacuum, i.e. it provides real final states of electrons and positrons from their initial virtual states. This means that the field which can dramatically change the final state of free beta-electron from its virtual state (in a nucleus) should be of the same order, i.e. of the order of E_{Sch}.

It is clear also, that there is not any opportunity now to measure the enhancement of beta decay. The main cause is the appearance of a number of electrons during the action of (strong) electric field on the atoms with nuclei considered for beta decayenhancement due to ionization by this electric field. The appeared beta-electrons are impossible to select from large quantity of plasma electrons with various energy.

2. ENHANCEMENT OF ORBITAL ELECTRONS CAPTURE BY NUCLEI

The enhancement of the orbital electronscapture, which is a process inverse to beta decay, has not yetdrawn so significant attention as beta decay, in particular because thiseffect has been known for a long time (for example, the rate ofthe $_7$Be $\rightarrow_7$Li process depends on the chemical bondinvolving the beryllium atom, see [9]). Furthermore,the theoretically and experimentally studied variety ofeffects on the wave-functions of orbital electrons wasvery wide. These are effects due to chemical bonds, highpressure [10], thermal effects from superconductivityand internal electric and magnetic fields of a medium,and plasma effects (see review [11]). The *K*-electroncapture was primarily considered. The possible enhancement did not exceed 0.01. The known papers (see [12, 13]) on the nuclear excitation at electron transitions of the corresponding atom are also close to the above works.

The electron capture from atomic shells higherthan the *K* shell is also well known [14, 15], includingthe capture of electrons with a nonzero orbital angularmomentum *l* [16, 17]. Namely, the latter process can be accelerated by the external electric field. In the case of theforbidden electron capture, a (bound electron–free neutrino) pair should compensate the change in thetotal angular momentum of the nucleus; i.e., it shouldbe either changed due to the captured orbital electronor carried out by the neutrino. The probability of thetransfer of the orbital angular momentum of the electron to the nucleus or the carrying out this momentum by the neutrino is determined by two factors. Thefirst factor is the relation between the nuclear radius,characteristic radius of the electron wave-function, andde Broglie wavelength of the neutrino. The probabilitythat the orbital angular momentum is carried out bythe free neutrino is determined by the ratio of thenuclear radius r_n to the de Broglie wavelength of theneutrino in a power of $2l$ (l is the carried out orbital angular momentum). At the same time, the probability of the transfer of the orbital angular momentumfrom the orbital electron to the nucleus is determinedby the ratio of the nuclear radius to the characteristicradius of the wave-function of the orbital electron in thesame power $2l$. The second factor is that the orbitalelectron already has the orbital angular momentum that canbe transferred to the nucleus, whereas the

emittedneutrino should collect this momentum in the decay process. According to the uncertainty relation, thisprocess occurs quite rapidly at high energies of theemitted neutrino ε_ν. Both of these factors are responsible for the above mentioned enhancement of the forbidden capture of the orbital electron in the *p*, *d*, etc. states. Thus, at $\varepsilon_\nu < 1$ MeV, the forbidden electron capture occurs primarily with the corresponding electrons in the *p*, *d*, etc. states; i.e., the capture occurs from the L_{III}, M_V, N_{VII}, etc. shells. A characteristic example is the electron capture by $_{205}$Pb nucleus, where the ratio of therates of *K* and *L* captures is estimated as 10^{-4} [17].

According to the theory of the capture of orbital electrons [18], the probability of the allowed andunique first, second, etc. forbidden captures is proportional to the square of the matrix element of the corresponding transition between the parent and daughternuclei, the square of the wave-function of the capturedelectron on the nucleus, the square of the neutrinoenergy for the allowed capture, the fourth power of theneutrino energy for the first forbidden capture, thesixth power of the neutrino energy for the second forbidden capture, etc., and the square of the Fermi constant of the weak interaction. The unique first and second forbidden transitions, which are determined byone nuclear matrix element, are of primary interest[2]. These electron captures are well known. One of themis the above mentioned first forbidden process in $_{205}$Pb. There are other examples the most interesting ofwhich are given below.

3. ACTION OF THE EXTERNAL ELECTRIC FIELD ON THE ELECTRON CAPTURE

Among the factors entering into the probability of the process, only the wave-function of the bound electron on the nucleus can be really changed by the external electric field, as discussed above in Sect. 1. Indeed, the external electric field polarizes the atom. As a result, the wave-functions of all the electrons of the atom are shifted with respect to their nucleus. This obviously can only reduce the wave-function of *s* electrons on the nucleus, because this wave-function is maximal on the nucleus. At the same time, the wave-functions of *p*, *d*, etc. electrons vanish at a certain point inside the nucleus. This is valid in both the Dirac description of singleelectron wave-functions in the atom and the simple Slater approximation [19]. The polarization shift can only increase the density of these electronic states. The enhancement of the electron capture is possible due to this increase in the electron density with the necessary orbital angular momentum on the nucleus.

In the current theory of beta processes [18], it iscommonly accepted to describe the single-electronstate by the relativistic Dirac equation (see, for example, [20]) with the self-consistent Hartree–Fock potential. Previously (until the 1960s inclusively), electron wave-functions were used in the mentioned Slater approximation [21], where the main inaccuracy was the disregard ofrelativistic effects. The Slater approximation is sufficient to demonstrate the possibility of the external electric field induced enhancement of electron capture, which is theaim of this work.

The possible effect is estimated as follows. For theinner electron shells (*K*, *L*, and, to certain extent, *M* shells) of atoms, the wave-functions for L_{III} and M_V subshells near the nucleus have the values

$$\Psi_{L_{III}} \cong \frac{Z_{L_{III}}^{\frac{5}{2}}}{4\sqrt{2\pi}\, r_B^{\frac{5}{2}}} r\cos\theta$$

$$\Psi_{M_V} \cong \frac{Z_{M_V}^{\frac{7}{2}}}{324\sqrt{2\pi} r_B^{\frac{7}{2}}} r^2\left(\cos^2\theta - \frac{1}{3}\right) \qquad (5)$$

Here, r_B is the Bohr radius, *r* is the distance from the center of the nucleus, Z_{LIII} and Z_{Mv} are the effective screened charges of the nucleus for the respective subshells, and θ is the polar angle. If the Dirac equationwith the screened Coulomb interaction potential isused, the factor r^l in the wave-functions given by, Eqs. (5) is replaced by the factor $r^{l'}$, where $l' = l - Z_{eff}e^2 / 2\hbar c(l+1)^2$ and Z_{eff} is the effective screenedcharge of nucleus. Even for the heaviest nuclei, the differencebetween *l* and *l'* for $l = 1$ is 4%. Only states with themagnetic quantum number $m_q = 0$ are of interest (seebelow).

Wave-functions in Eq. (1) are written for the pure Coulomb interaction of the electron with the nucleus inthe Schrödinger equation. However, their hyperfineinteraction leads to the appearance of a certain constant addition to the wave-function on the nucleusgiven by Eqs. (5). It is easy to show that this additionfor $l = 1$ and 2, i.e., for the first and second forbiddenelectron captures does not affect the external electric field induced enhancement of the electron captures under consideration and the hyperfine interaction reducesthe enhancement of the third forbidden capture, mostly in heavy nuclei.

The atom in the electric field is polarized. In thesingleelectron Slater approximation, the dipolemoment induced by the external electric field E is estimated from the Stark shift of the corresponding levelas follows:

$$d_{nm_q} = \frac{n^4}{8}\left(17n^3 - 9m_q^2 + 19\right)\frac{Er_B^3}{z_{eff}^4} \tag{6}$$

where $Z_{eff} = Z_{LIII}$ for the L_{III} subshell, $Z_{eff} = Z_{Mv}$ for the M_V subshell, etc., n is the principal quantum number, and $\boldsymbol{L}_{nmq} = \boldsymbol{d}_{nmq}/e$ is the characteristic polarization shift of the wave-function. In order to estimate perturbed wave-functions, it is necessary to perform vector summation as $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{L}_{nmq}$. Since atoms are oriented chaotically, the result should be averaged over orientations. The time averaging of the resulting effect is also necessary.

The electric field amplitudes necessary for the enhancement of the electron capture are bounded frombelow by the condition $L_{nmq} > r_n$. There is also an upper limit, which is determined by the necessity of the existence of the bound electron with a certain orbital angular momentum. Perturbed wave-functions on the nucleus can be easily estimated by substituting L_{nmq} for$r$ in Eqs. (5) and performing the appropriate averaging procedures. The binding energies of the electrons in the inner atomic shells are in the keV range. Corresponding frequencies of these electrons are many orders of magnitude higher than the frequency of laser radiation and characteristic frequencies of plasma microfields. Hence, the laser radiation as well as plasma microfield can be considered as quasi-stationary perturbation. It is convenient to represent the expression for the value of L_{nmq} in terms of the electric field in the hydrogen atom that is created by the proton at a distance of $r_B (E_{at} = e/r_B^2 = 5.5\cdot10^9$ V/cm). Then, according to Eq. (6) with $n = 2$ and $m_q = 0$, the factor α_1 of the enhancement of the first forbidden electron capture is proportional to the square of an external electric field E, ratio of the squares of the Bohr radius to the nuclear radius, and inversely proportional to the eighth power of the effective charge of the nucleus. For the characteristic values $Z_{LIII} \sim 33$–70 for the elements with $Z \sim 50$–92 and $r_n \sim (5.5$–$7.5)f^3$, α_1 is estimated in the range $(0.1$– $0.0001)\ E^2/E_{at}^2$. As will be shown by correct perturbation theory calculations, this value is strongly overestimated. Nevertheless, it also indicates that the enhancement of the process at $E^2/E_{at}^2 \sim 1$ is small and very high amplitudes of external electric fields are necessary for a noticeable effect. This estimation also means that the Stark shift of X-rays [22] is small as follows: $\langle d_{20}E\rangle \sim (0.0036$–$0.00018)\ E^2/E_{at}^2$ eV. For this reason, a change in the electron capture rate due to a change in the energy of the emitted neutrino can be neglected here.

To estimate the enhancement of the second forbidden electron capture, Eq. (6) with $n = 3$ and $m_q = 0$ should be used. The factor α_2 of the enhancement of the second forbidden electron capture is proportional tothe fourth powers of external electric field amplitude, the ratio of the fourth powers of the Bohr radius to the nuclear radius, and the sixteenth power of the inverse effective charge of the nucleus. In this case, the characteristic screened charges are smaller, $Z_{Mv} \sim 18$–47 for elements with $Z \sim 50$–92 and $r_n \sim (5.5$–$7.2)f$. The quantity α_2 is estimated as $(10^2$–$10^8)$ $(E^2/E_{at}^2)^2$. It will be shown that this value is also overestimated, but external fields amplitudes necessary for the tenfold enhancement of the electron capture are in any case comparable to or smaller than E_{at}. The Stark shift of X-ray lines will be about $(0.0077$–$0.36)E^2/E_{at}^2$ eV. A change in the energy of the emitted neutrino can be neglected. The enhancement of the third forbidden electron capture is obviously proportional to the sixth power of E and the effect should be observed already at standard electric fields of high-voltages facility (even taking into account the above mentioned overestimation).

The correct calculation of the effect of the captureof p, d, etc. electrons should include the calculation of the wave-functions of these electrons near the nucleus taking into account the quasi-stationary electric field. Perturbation theory can be used because the external electric field under the conditions under consideration can be treated as relatively small at $E < Z_{eff}^3 E_{at} \sim (10^4$–$10^5)\ E_{at}$ (non-ultrarelativistic case).

4. WAVE-FUNCTIONS OF ELECTRONSNEAR THE NUCLEUS IN THE EXTERNALELECTRIC FIELD

The effect of the external electric field on the single-particle hydrogen-like Slater wave-functions of electrons can be considered using Heisenberg–Schrödinger perturbation theory. Only the wave-functions near the nucleus, i.e., for r values about several r_n are finally necessary. The Schrödinger equation with the effective screened Coulomb charge and external electric field can be written in the form

$$\Delta\Psi + \frac{2m}{\hbar^2}\left(\varepsilon + \frac{z_{eff}e^2}{r} - eEr\right)\Psi = 0 \tag{7}$$

Here, Ψ and ε are the wave-function and energy of the corresponding electronic state, respectively. As usual, in problems with the external electric field, it is reasonable to transform of Eq. (7) to the parabolic coordinates (see, e.g., [19] or [23], the latter containing the best description). The variables are separated in coordinates ξ, η, and φ. The normalized wave-function characterized by the parabolic quantum numbers n_1, n_2, and m_q is written as follows:

$$\Psi_{n_1 n_2 m_q} = \sqrt{2}\rho^{\frac{3}{2}} f_{n_1 m_q}(\rho\xi)\, f_{n_2 m_q}(\rho\eta)\frac{e^{im_q\varphi}}{\sqrt{2\pi}} \tag{8}$$

Unlike [23], the value $\rho = Z_{eff}/nr_B$ is nowinverse effective Bohr radius, $n = n_1 + n_2 + |m_q| + 1$ is the principal quantum number, and the functions f_{n1mq} and f_{n2mq} satisfy the equations

$$\frac{d}{d\xi}\left(\xi\frac{d}{d\xi}\right) f_{n_1 m_q} + \left(\frac{m\varepsilon}{2\hbar^2}\xi - \frac{meE}{4\hbar^2}\xi^2 - \frac{m_q^2}{4\xi}\right) f_{n_1 m_q} = -\beta_1 f_{n_1 m_q}$$

$$\frac{d}{d\eta}\left(\eta\frac{d}{d\eta}\right) f_{n_2 m_q} + \left(\frac{m\varepsilon}{2\hbar^2}\eta + \frac{meE}{4\hbar^2}\eta^2 - \frac{m_q^2}{4\eta}\right) f_{n_2 m_q} = -\beta_2 f_{n_2 m_q} \tag{9}$$

where $\beta_1 + \beta_2 = Z_{eff}e^2m/\hbar^2$, $\beta_1 = [n_1 + (|m_q| + 1)/2]\, Z_{eff}e^2m/n\hbar^2$, and $\beta_2 = [n_2 + (|m_q| + 1)/2]Z_{eff}e^2m/n\hbar^2$.

As it was mentioned above, the term with the field in Eqs. (9) can be considered as a small perturbation for external electric field amplitudes satisfying the inequality $E < Z_{eff}^3 E_{at}$, i.e., for all existing facilities. In this case, the first order correction to the wave function $f^{(1)}_{n_i m_q}(x)$ [24] is given by the expression

$$f^{(1)}_{n_i m_q}(x) = \frac{E}{4Z_{eff}e\rho^2}\Sigma_{n_i \neq n_j}\frac{\langle x\rangle^2_{n_i n_j}}{n_i - n_j} f^{(0)}_{n_i m_q}(x) \tag{10}$$

where $f^{(0)}_{n_i m_q}(x)$ is the solution of Eqs. (9) with $E = 0$ andthe matrix element $\langle x\rangle^2_{n_i n_j}$ is the average of the squareof ξ or η over the unperturbed wavefunction $f^{(0)}_{n_i m_q}(x)$ (the values of these matrix elements can be found in[23]). The second order correction $f^{(2)}_{n_i m_q}(x)$ [24] is givenby the expression

$$f^{(2)}_{n_i m_q}(x) = \frac{E^2}{16Z_{eff}^2e^2\rho^4}\left[\Sigma_{n_i \neq n_k}\Sigma_{n_i \neq n_l}\frac{\langle x\rangle^2_{n_i n_l}}{n_i - n_l}\frac{\langle x\rangle^2_{n_i n_k}}{n_k - n_i} f^{(0)}_{n_k m_q}(x) - \Sigma_{n_i \neq n_s}\frac{\langle x\rangle^2_{n_i n_l}\langle x\rangle^2_{n_i n_s}}{(n_i - n_s)^2} f^{(0)}_{n_k m_q}(x) - \frac{1}{2}\Sigma_{n_i \neq n_s}\frac{\langle x\rangle^2_{n_i n_s}}{(n_i - n_s)^2} f^{(0)}_{n_i m_q}(x)\right] \tag{11}$$

The first order correction for wave-function (8) has the form

$$\Psi^{(1)}_{n_1 n_2 m_q} = \sqrt{2}\rho^{3/2}\left[f^{(1)}_{n_1 m_q}(\rho\xi)\, f^{(0)}_{n_2 m_q}(\rho\eta) + f^{(0)}_{n_1 m_q}(\rho\xi)\, f^{(1)}_{n_2 m_q}(\rho\eta)\right]\frac{e^{im_q\varphi}}{\sqrt{2\pi}} \tag{12}$$

and the second order correction is given by the expression

$$\Psi^{(2)}_{n_1 n_2 m_q} = \sqrt{2}\rho^{3/2}\left[f^{(2)}_{n_1 m_q}(\rho\xi)\, f^{(0)}_{n_2 m_q}(\rho\eta) + f^{(1)}_{n_1 m_q}(\rho\xi)\, f^{(1)}_{n_2 m_q}(\rho\eta) + f^{(0)}_{n_1 m_q}(\rho\xi)\, f^{(2)}_{n_2 m_q}(\rho\eta)\right]\frac{e^{im_q\varphi}}{\sqrt{2\pi}} \tag{13}$$

The behavior of electric fieldperturbed wave-functions (5) in the spherical coordinates near the origin is of interest. Small r values correspond to small ξ and η values; therefore, the functions $f_{n_1 m_q}$ and $f_{n_2 m_q}$ can be expanded in the Taylor series. The electric fieldinduced polarization of the wave-function exists for states with $m_q = 0$ only[4]. Then,

$$\Psi^{(1)}_{n_1 n_2 0}(\xi, \eta \sim 0) = \frac{3(n_1 - n_2)n^{1/2} E r_B^{1/2}}{8\sqrt{\pi} Z_{eff}^{3/2} e} \tag{14}$$

i.e., the first order correction is proportional to the difference between parabolic quantum numbers. Whenthese numbers are equal to each other, this correctionvanishes [23, 24]. The second order correction forwave-function (8) has the form

$$\Psi^{(2)}_{n_1 n_2 0}(\xi, \eta \sim 0) = \frac{n^{5/2} E^2 r_B^{5/2}}{8\sqrt{\pi} Z_{eff}^{5/2} e^2}\left(n_1^3 + n_2^3 - n_1^2 - n_2^2 + 2n_1 + 2n_2 - \frac{3}{2}\right) \tag{15}$$

It is nonzero even when the parabolic quantum numbers are the same.

Now, wave-function (5) in the external electric field will be calculated. The exact hydrogen-like wave-functions in the spherical coordinates are linear combinations of the wave-functions in the parabolic coordinates [23],

$$\Psi_{2,1,0}(r, \theta, 0) = \frac{1}{2\sqrt{2}}\left[\Psi_{1,0,0}(\xi, \eta) + \Psi_{0,1,0}(\xi, \eta)\right] \tag{16}$$

$$\Psi_{3,2,0}(r, \theta, 0) = \frac{1}{\sqrt{2}}\left[\Psi_{2,0,0}(\xi, \eta) + \Psi_{0,2,0}(\xi, \eta) - 2\Psi_{1,1,0}(\xi, \eta)\right] \tag{17}$$

After the substitution of Eq. (12) into Eq. (14), it is seen that the first order correction to the first wave-function near the nucleus in Eqs. (5) is nonzero and

$$\Psi_{2,1,0}(r \to 0, \theta, 0) \cong \frac{Z_{L_{III}}^{\frac{5}{2}}}{4\sqrt{2\pi} r_B^{\frac{5}{2}}} r\cos\theta + \frac{3E\sqrt{3r_B}}{8\sqrt{\pi}\, Z_{L_{III}}^{\frac{3}{2}}}\cos(\theta - \gamma) \tag{18}$$

where γ is the angle between the vector of the electric field and the polar angle of the wave-function. A similar calculation of the first order correction to the second wave-function at zero in Eqs. (5) indicates that it is zero. Consequently, one should take into account the second order correction, which has the form

$$\Psi_{3,2,0}(r \to 0, \theta, 0) \cong \frac{\sqrt{3} Z_{M_V}^{\frac{7}{2}}}{324\sqrt{2\pi} r_B^{\frac{7}{2}}} r^2\left(\cos^2\theta - \frac{1}{3}\right) + \frac{9\sqrt{3}E^2 r_B^{5/2}}{2\sqrt{\pi} Z_{M_V}^{\frac{9}{2}} e^2}\left[\cos^2(\theta - \gamma) - \frac{1}{3}\right]^2 \tag{19}$$

For the third forbidden electron capture, it is necessary to calculate the third order correction to the wave-function $\Psi_{4,3,0}(r \to 0, \theta, 0)$.

As was mentioned above, in order to obtain theenhancement factor of the electron capture α, it is necessary to average the absolute values of the squares of the wave-functions given by Eqs. (16) and (17) over time and the orientation of the electric vector of the external field and to calculate the integral over the volume of the nucleus. Then, the resulting value should be divided by the integral of unperturbed wave-functions over the volume of the nucleus. Large α values for which the polarization shift of the wave-function is (much) larger than r_n are naturally of interest. Thus, for the first forbidden electron capture,

$$\alpha_1 = \alpha_{L_{III}} \approx \frac{25}{4Z^8_{L_{III}}}\left(\frac{r_B}{r_n}\right)^2 \frac{E^2}{E^2_{at}} \tag{20}$$

This result is approximately two orders of magnitude smaller than the estimate obtained using L_{210} (see Eq. (6)).

The enhancement factor of the second forbiddenelectron capture under similar conditions has the form

$$\alpha_2 = \alpha_{M_V} \approx \frac{3^{15}}{8Z^{16}_{M_V}}\left(\frac{r_B}{r_n}\right)^4 \frac{E^4}{E^4_{at}} \tag{21}$$

This result is five orders of magnitude smaller than the estimation obtained using Eq. (6).

5. POSSIBILITIES OF EXPERIMENTALOBSERVATION

There are 3 possible types of experiments to detect the effect of orbital electron capture enhancement by external electric field: using superstrong laser fields [25], high-voltage static electric fields, and (dense) plasma (micro)fields. The laser radiation intensities necessary for theenhancement of the first forbidden electron capture are high enough. At present, they are implemented in short superstrong laser pulses shorter than 1 ps with a low repetition frequency. Nevertheless, the observation of the enhancement of characteristic X-ray radiation with transitions from the L_{III} shell is apparentlypossible at least in the X-ray photon count regime. The significant ionization of atoms under investigation will be accompanied by the possible enhancement of the electron capture.The decay of the long-life (1.53×10^7 yr) lead isotope, $_{205}\mathrm{Pb} \to {}_{205}\mathrm{Tl}$, is accompanied by a change in the total angular momentum of the nucleus by two unitsand in parity (this is the unique first forbidden electron capture). The enhancement specified by Eq. (20) is $\alpha_1 \approx 1.7 \times 10^{-6} E^2/E_{at}^{\ 2} = 1.7 \times 10^{-6} I_{l0}/I_0$, $I_0 = cE_{at}^{\ 2}/8\pi$, $I_{l0} = cE^2/8\pi$ – maximal intensity of laser radiation (pulse). A significant enhancement of this electron capture ($\alpha_1 \sim 10$) requires the laser radiationintensity $\sim 10^{23}$ W/cm^2. These intensities have not yet been available. Another similar process is the electron capture in the long-life isotope $_{81}$Kr (2.29×10^5 yr [15]). For the $_{81}\mathrm{Kr} \to {}_{81}\mathrm{Br}$ decay, $\alpha_1 \sim 0.35E^2/E_{at}^{\ 2}$. An enhancement factor of 10 is achieved already at laser intensities $I_{l0} \sim 10^{18}$ W/cm^2.

The experimental possibilities of the observation ofthe second forbidden electron capture enhancement in the laser field are much more obvious, becausethe effective charge is smaller and polarizability ishigher. For the second forbidden electron capture in,e.g., the133Ba isotope (with a half-life of 10.51 yr), $_{133}\mathrm{Ba} \to {}_{133}\mathrm{Cs}$, $\alpha_2 \sim 300(I_{l0}/I_0)^2$, i.e., $\alpha_2 \sim 10$ at $I_{l0} \sim 10^{16}$ W/cm^2.

Processes involving stable nuclei are of most interest. For the second forbidden electron capture, $_{123}\mathrm{Te} \to {}_{123}\mathrm{Sb}$, $\alpha_2 \sim 1000(I_{l0}/I_0)^2$, i.e., $\alpha_2 \sim 10$ at $I_{l0} \sim 10^{15}$ W/cm^2. Up to now, this process was interpreted as *K*-electron capture according to radiation with a photon energy of about 28 keV, which corresponds to the $K\alpha$ line of $_{123}$Sb [26] with a lifetime longer than 10^{13} yr. This value was also presented in [15]. However recently, more accurate studies indicate that this radiation is absent and the $_{123}\mathrm{Te} \to {}_{123}\mathrm{Sb}$ decay cannot be treated as *K*-electron capture [27]. The restriction $t_{1/2} \geq 10^{17}$ yr was obtained for the half-life [27].

Laser intensities I_{l0} larger than 10^{12} W/cm^{2}can be realized in the above superstrong laser pulses experiments only. The opportunity to realize this enhancement in a permanent laser field is much more usable.Now, the highest

available permanent laser power is about 100 kW, which corresponds to the laser intensity 10^{11}- 10^{12} W/cm^2 under sharp focusing. The same intensities are required for laser breakdown (it is not desired in this case); therefore, isotopes for enhancement in such fields canbe founded among light nucleus with the second forbidden capture. The same can be said for non-laser experiments.

Indeed, the breakdown static electric field for glasses and (especially) fused silica is up to 10^9 V/cm. Some admixture of an investigated isotope (up to several percent by mass, see, for example, data for Nd-glasses [22]) provides necessary number of worked atoms. For example, the process of the second forbidden capture $_{54}Mn \rightarrow {}_{54}Cr$ (life-time of $_{54}$Mn is 312 d) has providedtheenhancement factor $\alpha_2 \sim 10$ already at $E< 100$ MV/cm on edges of experimental piece of glass or fused silica (note that the field inside these dielectrics is about 10 times less, it is used for above α_2). It looks that the enhancement of the most part of the third forbidden orbital electron captures in various isotopes can be conducted namely in these high-voltage experiments. Consequently, these experiments can be conducted in a permanent laser field.

6. ENHANCEMENT OF FORBIDDEN ELECTRON CAPTURES IN PLASMAS

There are several mechanisms to accelerate an electron capture in plasma. The first trivial mechanisms arises due to "compression" of "electron orbits" in ions (and the real Bohr radius of corresponding electron state) with respect to atomic "orbits". Indeed, the Bohr radius of *K*-electron in hydrogen-like ion is inversely proportional to real ion charge *Z*, while the Bohr radius for Slater *K*-electron state is inversely proportional to Z_{eff}. This Z_{eff} can be extracted from the X-ray spectra; in all cases $Z_{eff}<Z$ due to screening of other electrons (in hydrogen-like atom, they absent). For example, for H-like U, $Z = 92$, for atomic U, $Z_{eff} = 81{,}5$ [22]. Since the enhancement factor in this case is proportional to $(Z/Z_{eff})^3$, the considered case with U gives the enhancement 1,44 even for *K*-captures. The same occurs for all forbidden orbital electron captures by nuclei. In fact, orbital electrons influences nucleonic states in nucleus. Therefore, the presence/absence of orbital electron changes these states and the nuclear matrix elements for transitions between mother and daughter nuclei in the process of electron capture. Thus, the final result of such plasma effect (the absence of outer atomic electrons on ions) may be different from the simple enhancement $(Z/Z_{eff})^3$. Of course, the most input to the effect is due to *K*-, *L*-, and *M*-electrons, thoughplasmas with ions missing these electrons is not simple to produce (and confine). Speaking about "simple" highly ionized plasmas, we return to the action of plasmas electric (micro) field on the process of electron capture by nuclei. This effect is in line with the action of an electric microfield on other elementary processes: ionization, recombination, nuclear fusion [28].

The action of plasma electric microfield on the process of a forbidden electron capture by nuclei reduces to above polarization of Slater wave-function of the considered electron by this microfield. To determine this action, one needs to average electron wave-functions over the microfield distribution function *W(E)*. Since real wave-functions of Eq.(7) drops with large *E*, it is possible to do this in any case. At the same time, it is useful to average directly the wave-functions near the nucleus presented in Eq. (5). On this way, there appear some difficulties. The "normal" highly-ionized plasma is very hot and closes to the ideal one, and the Holtsmark approximation [29] for *W(E)* looks useful. The first averaged wave-function (5) gives $<E>_H \approx 8{,}8en_i^{2/3}(Z_i+Z_i^{2/3})$, Z_i is the ion charge (we consider simplified plasma model with one ionic species). The averaging of E^2 in the second wave-function (5) over Holtsmark distribution encounters the divergence, the same is for E^3, etc. To provide an estimate of the capture effect, one needs to take into account the real electric field created by plasma charges on the considered ion. This field cannot be larger than $E_{max} = Z_i e/r_i^2$ (r_i is the "outer" radius of the ion connected with the principal quantum number of the ion valence electron). In this case, Holtsmark distribution becomes truncated and belongs to the class of truncated Levy distributions (see, for example[30]). In this case, a good estimate for $<E^2>$ is the value $9.9E_{max}^{1/2} e^{3/2} n_i (Z_i+Z_i^{2/3})^{3/2}$. Thus, to determine the plasma enhancement factors (20) and (21), one needs to replace $<E>_H$ instead of *E* in Eqs. (20) and $<E^2>$instead of E^2 in Eqs. (21).

For the above first forbidden decay of $_{81}Kr \rightarrow {}_{81}Br$ in highly-ionized plasma of Ne-like Kr, $\alpha_1 \sim 10$ already at $n_i \sim 5\cdot 10^{21}$ cm^{-3}, which is about two hundred atmospheric pressures of Kr under room temperatures. Thus the above concentrated Ne-like plasma of $_{81}$Kr will have the life-time about 30 days with respect to the electron capture. Note that this Ne-like plasma requires temperatures of the order of 1 keV.

For the second forbidden decay of $_{123}$Te$\rightarrow_{123}$Sb in Ni-like plasma of $_{123}$Te, $\alpha_2 \sim 10$ already at $n_i \sim 10^{19}$ cm^{-3}. Such plasma can be confined during long time inside a standard gas dynamic trap since the temperature of this plasma is less than 1 keV. Thus, the enhancement of forbidden captures of orbital electron by nuclei in highly-ionized plasma has good enough experimental perspectives, especially for plasmas consisted of ions suitable for the second forbidden capture by their nuclei.

7. ENHANCEMENT OF NEUTRINOLESS DOUBLE CAPTURE OF ORBITAL ELECTRONS

Let uspropose some physical considerations regarding the possibility to achieve enhancementofa significant (thus far hypothetical) neutrinoless double capture of orbital electrons by nucleiunder the action of a strong electric field (including a laser field) on the atoms. Such process hypothesized as early as 1955 [31] can be realized if a neutrino coincides with its antiparticle (E.Majorana's hypothesis). In this case, two consecutive electron capture processes need to be summarized for a nucleus of mass number $A + 2$:

$$\begin{gathered}[(A+2)+e\rightarrow(A+1)+\nu_e] \\ \underline{+[(A+1)+e+\overline{\nu_e}\rightarrow A+\gamma} \\ [(A+2)+2e\rightarrow A+\gamma]\end{gathered} \tag{22}$$

Formally, in this case the neutrino of the first process and the antineutrino of the second one cancel each other (physically, the neutrino is emitted in the former process and absorbed in the latter). The two events are integrated into the resulting process with the emission of a gamma-quantum.

There are several possibilities to observe such transitions. The first type of them occur between stable nuclei $0^+ \rightarrow 0^+$, there are a total of 12 such pairs with pure electron capture without positron emission [15]. The second opportunity will be discussed later.

It is clear that the time between the first and the second processes is not too large, at least shorter than the neutrino flight of time in the nucleus of interest, and the distance traveled by the neutrino is shorter than the nucleardiameter. This fact should be accounted for the value of the nuclear matrix element transition between the parent (A + 2) and daughter (A) nuclei; it appears to be lower for transitions with a smaller intermediate nuclear moment. Indeed, the uncertainty relation for the values of angle φ and angular momentum L_z [20]

$$\left\langle (\Delta L_z)^2 \right\rangle \left\langle (\Delta\varphi)^2 \right\rangle \geq \frac{\hbar^2}{4} \tag{23}$$

at small angular displacements Δφ (the spatial displacements of protons that captured an electron inside the nucleus being small too) determines rather large values of ΔL_z associated with the first capture, i.e. in the intermediate nucleus A + 1. For this reason, the above mentioned nuclear matrix element has to be (much) larger for such neutrinoless double captures in which capture by the intermediate nucleus is forbidden. It is clear, however, that the wave function of the electron being captured on the nucleus is smallerfor forbidden captures by a factor of $Z_{eff} r_n / r_B$in the correspondingpower, and the result for the double forbidden and allowed capture rates is roughly equal. At least, the difference is not so large as for single-electron captures [18, 21]. When the wave function of the captured electron with the non-zero orbital quantum number is shifted by an electric field with respect to the nucleus, the value of this function on the nucleus increases which leads to the enhancement of the double electron capture. In transitions between nuclei $0^+ \rightarrow 0^+$, the enhancement factor is simply the product of corresponding enhancements of the forbidden single-electron captures (20), (21).

8. NEUTRINOLESS DOUBLE CAPTURE IN THE RESONANT STATE OF DAUGHTER NUCLEI IN PLASMA AND POSSIBILITIES OF ITS EXPERIMENTAL REALIZATION

The excess energy in neutrinoless double capture has to be eliminated by a γ-quantum with the appropriate energy. If the daughter nucleus lacks resonance levels in close proximity to the ground state of mother nucleus (minus the

binding energy of the electrons being captured), the main process by which such γ-quanta are generated is bremsstrahlung [32]. The presence of such closelylocated excited levels would sharply increase the probability of double electron capture because it might occur in the resonance state with subsequent resonantradiation of the corresponding γ-quantum. The calculation of such resonant processes for certain nuclei is reported in Refs [33-36]; it is proposed to use synchrotron radiation to realize induced transitions into the excited state of the daughter nucleus [37]. The resonance parameter F in the probability of double electron capture into the excited state has the form [38]

$$F = \frac{\Gamma_{2h}}{\Delta^2 + \Gamma_{2h}^2 / 4} \tag{24}$$

where $\Delta = Q\text{-}B_{2h}\text{-}\varepsilon_\gamma$, B_{2h} and Γ_{2h} are the energy and the width respectively of a double electron hole in the atom's electron shell of the daughter nucleus, Q is the difference between the binding energies of the parent and daughter nuclei, ε_γ is the energy of the excited level in the daughter nucleus.

Among the known nuclei, $_{74}$Se has one of the lowest Δ: for electron capture $_{74}\text{Se}\rightarrow{}_{74}$Gefrom L_{III} and L_I shells, Q = 1209,7 keV [33, 39] with the best accuracy +/-0,25 keV [39], ε_γ = 1204,205+/–0,007 keV [39], B_{2h} = 1,371 + 1,579 = 2,950+/ – 0,001 keV [22], and Δ = 2,55 keV[5] (see Fig. 1). Initially, the first forbidden capture by the virtual $_{74}$As nucleus ground state 2^- occurs; it is followed by the allowed capture into the excited state 2^+ of $_{74}$Ge. According to Ref [33], the upper experimental estimation of the half-lifeat such detuning Δ_{Se} (in the first capture from the L_{II} but not L_{III} shell) is $0{,}55\cdot10^{19}$ years. The above reasoning leads to the conclusion that the double capture from the L_I and L_{III} shells must be faster. There is a set of theoretical estimations of this half-life, which depends (inversely proportional to the square) on the mass of Majorana neutrino, it provides much smaller value of this half-life with reasonable value of above neutrino mass. The accuracy of such estimates is not high yet, thus it is better to navigate to the above experimental estimations even for the lower limit of them.

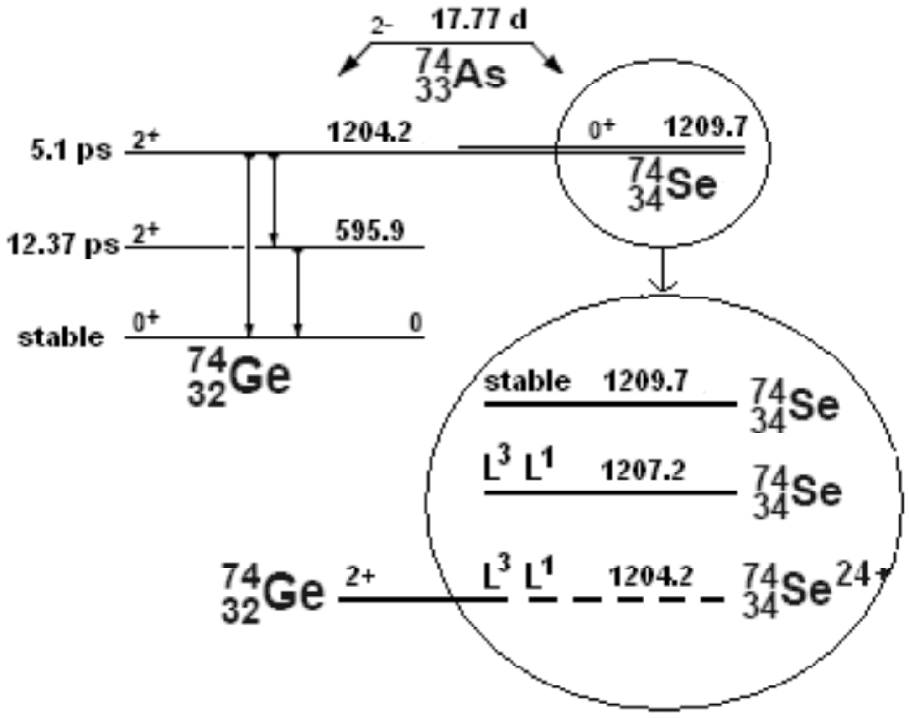

Figure 1: Schematic of energy layers in the neutrinoless capture $_{74}\text{Se}\rightarrow{}_{74}$Ge. Left: lower excited levels of $_{74}$Ge and the corresponding half-lives. The ellipse encompasses energy layers of the $_{74}$Se ground state, double holes in the L_I and L_{III} electron shells of a $_{74}$Se atom and a neonlike $_{74}\text{Se}^{24+}$ ion. The 1204.2 → ground state transition in $_{74}$Ge is roughly twice less probable than the 1204.2 → 595.9 transition [15] (all energies are expressed in keV)

During the following atomicionizations, the absolute value of the electron binding energy in the remaining shells becomeshigher nucleus charge shielding decreases. Above that, the ionization energy during transition between electronic shells undergoes astrong jump (ionization energy of neonlike Se is 2542 eV compared with 1036.3 eV in the preceding Se ion). This fact was used in designing X-ray lasers [40, 41]. The electron binding energy in the L_{III} shell of neonlike $_{74}Se^{24+}$ approximately equals the ionization energy, 2542 eV, or less; the electron binding energyin the L_I shell can be estimated as 2900 eV (accuracy of this estimation is about 25 eV, we used data of X-ray spectra as more responsible), therefore, $\Delta_{Se}{}^{24+}$ is not larger than 70 eV, practically this value should be smaller due to the presence of all electrons in the L_{III} shell of $_{74}Se^{24+}$ (see the Fig. 1). The value of Γ_{2h} in the $_{74}Se^{24+}$ ion is about Stark width ~ several eV and being different (~ 1.5 times higher) from the atomic value of Γ_{2h}, this fact should be taken into account in the enhancement factor.

Thus, there are two enhancement factors of neutrinoless double capture of electrons from the L_{III} and L_I shells of $_{74}Se^{24+}$ in plasma [42]. The strongestof them is bringing quantity (24) into resonance as a result of the Stark shift of the inner electron energy levels in the ion as compared with those in the atom. In this case, the enhancement of the first forbidden capture from the L_{III} shell by a virtual nucleus by the action of plasma microfield (in the absence of enhancement of the capture from the shell L_I (see above)) is not too high. The cumulativeenhancementfactor α_1^{res} can be obtained by multiplying the resonance factor ratio (24) by α_1. In this case, if

$$\frac{25}{4Z^8_{L_{III}}}\left(\frac{r_B}{r_n}\right)^2\frac{E^2}{E^2_{at}}>1$$

then,

$$\alpha_1^{res}\approx\frac{25}{4Z^8_{L_{III}}}\left(\frac{r_B}{r_n}\right)^2\frac{E^2}{E^2_{at}}\frac{\Delta^2\Gamma^i_{2h}}{\Delta^2_{at}\Gamma_{2h}} \qquad (25a)$$

In the opposite case,

$$\alpha_1^{res}\approx\frac{\Delta^2\Gamma^i_{2h}}{\Delta^2_i\Gamma_{2h}}(25b) \qquad (25b)$$

Substituting parameters L_I and L_{III} of $_{74}$Se and Γ_{2h} ~ 1 eVand propose non-solid-state plasma densities that provides the absence of enhancement of electron capture from the L_{III} shell by plasma microfield (the case (25b) is realized), one get the minimal yield of α_1^{res} ~ 10^4 and therefore the time of such process in the laser field of the specified intensity ~ $5\cdot10^{14}$ years. Let us remind that we take the minimal experimental estimate for the $_{74}$Se life-time.

A hot dense plasma of $_{74}$Se can be generated in different ways [40, 41]. One way is to confine it in a trap created by pulsed beams of one or several CO_2-lasers (see [43, 44] for the description of traps)[6], or in gas-dynamic trap (see, for example [45]). When a plasma confinement system confines the number of ions equivalent to 1 g of $_{74}$Se, the minimum time of an experiment needed to detect γ-quanta from an emitting $_{74}$Ge nucleus with energies 608.35 and 595.85 keV may be as short as a few seconds. Note that the natural abundance of $_{74}$Se in Se is 0,9%, thus $_{74}$Se should be enriched before experiments.

The another opportunity to accelerate double neutrinoless electron capture in in plasma of $0^+\rightarrow0^+$ stable nuclei is connected with the ion of $_{168}$Yb. The *Q*-value for double neutrinoless capture $_{168}$Yb → $_{168}$Er is 1422,1 keV [15][7]. There is wide 4^+ level of 1411,098 keV with the set of characteristic gamma-lines to lower levels in the $_{168}$Er nucleus. In case of proper ionization, all remained levels drop under the change of the ionization potential. For example, the binding energy of electrons in the M_V shell of $_{168}$Yb atom is about 1531 eV [22], and their binding energy in Ni-like $_{168}Yb^{42+}$ is about 3493.5 eV [46]. Thus the binding electron energy can vary in wide range, and proper force of ionization can be selected. The properly ionized$_{168}$Yb has a wide set of electrons with binding

energy larger than 5,5 keV: the pair of such electrons may provide the resonance character of double neutrinoles capture. The accuracy to "hit" to resonance may be now better than 70 eV like for Ne-like $_{74}$Se. The natural abundance of $_{168}$Yb in Yb is 0,13%; the enrichment of the Yb is much more complicated than the Se one.

Probable neutrinoless double electron captures in $0^+ \to 0^+$ stable nuclei do not complete the list of opportunities to accelerate the neutrinoless double electron capture. It is possible to impress "double" resonance transition from an electron capture active nucleus with comparably long-life through real excited state of stable nucleus to an excited state of beta-radioactive nucleus. No coincidence of such resonance levels exists in atoms, but there are perspective triples of ions with the consequent nuclei. Of course, the "exact" hit to double resonance is impossible, but situation where one resonance (in final nucleus) is "good", and another resonance is "medium" are reliable. For example, there is the triple $_{194}$Hg(life-time 444y)$\to_{194}$Au $\to$ $_{194}$Pt, there is the level 2^- 35,19 keV in $_{194}$Au ($Q_{\text{Hg-Au}}$ = 40 keV), and there is the level 2^+ 2537 keV in $_{194}$Pt ($Q_{\text{Hg-Pt}}$ = 2542 keV). Proper ionization of $_{194}$Hg may compensate $\Delta_{\text{Hg-Pt}}$ = 5 keV, provide the "good" hit to the above resonance level of $_{194}$Pt, and strongly "improve" the resonance with the level in $_{194}$Au. The final enhancement factor may be of the order of 10^7. Since the initial rate of such probable double neutrinoless electron capture starting the initially unstable nucleus (with respect to electron capture!) may be expected (much) faster than for above stable nuclei, the observation of double neutrinoless orbital electron capture as well as measurement of Majorana neutrino mass becomes more reliable.

9. CONCLUSIONS

External electric fields necessary for enhancement of beta decay are very high and out of the range of possible realizations up to now. Some opportunities to generate such high fields and produce some beta-processes (not only beta decay) is connected with forthcoming superstrong laser pulse facilities.

At the same time, necessary magnitudes of external electric fields to accelerate forbidden captures of orbital electrons by nuclei are much smaller. The external fields needed to accelerate the first forbidden capture of electrons are relatively high yet: at present, they are realized in short superpower laser pulses with duration less than 1 ps in duration at a low repetition rate. Nevertheless, observation of the enhancement of characteristic X-ray radiation with transitions from the L_{III} shell appears possible even if in the X-ray photon counting regime.Enhancement of the second forbidden capture is easier to realize. The necessary external fields can be achieved even in high-voltage experiments with a permanent electric field of 10-100 MV/cm, i.e. they are smaller than the breakdownfield voltage of many pure dielectrics containing nuclei of interest. The enhancement of forbidden orbital electron capture is possible in (dense) plasmas by the action of an electric microfield also.

Finally, considerable enhancement of the neutrinoless double capture of electrons as well as verification of Majorana's hypothesis of the nature of neutrino is possible in an experiment with plasmas consisting of electrons and ions ionized to the proper structure.Such ionization changes the binding energy of remained electron and provides the better "hit" to resonance with the excited states of daughter nucleus. It can be not only processes between stable nuclei, but even between long-life unstable nuclei with the stable nucleus with real resonance level as an intermediate one.

The work was supported by the grant of A. von Humboldt Foundation and the program of Russian Academy of Sciences.

Notes

1. Numerous incompletely accurate works, where the acceleration of beta decay by comparatively moderate external fields was predicted, are not mentioned here.
2. To simplify the consideration of the effect of the electric field on the electron capture, the exchange and overlapping effects are disregarded in this work. Their effect is noticeable, but not decisive [18].
3. The relation $r_n = 1.2A^{1/3}f$ is used, where A is the mass number of the isotope [18].
4. The effect of a static uniform electric field on single electron wave-functions with $m_q \neq 0$ is different. Since these wave-functions are nonzero in the axisymmetric regions and change sign, the zero of the wave-function is not displaced from the point $r = 0$. Only the electricfieldinduced change in the coefficient of rl for the wave-function with the orbital angular momentum l occurs in Eq. (5).

5. Previous data [15] gave 2,25 keV with the accuracy of about one keV.
6. Note that the high-density plasma of 74Se with $n_i > 5 \cdot 10^{21}$ cm^{-3} (~ 1/10 from solid-state density) acquires the acceleration of the first forbidden capture to the virtual nucleus, and final enhancement will be expressed by (25b).
7. Data [15] show the 0+ level of 1422,1 keV in daughter nucleus $_{168}$Er. Does it mean that the resonant process in $_{168}$Yb is possible in Nature?

References

[1] A. I. Nikishov, V. I. Ritus. *Zh. Eksp. Teor. Fiz.* **46**, 1768, (1964).

[2] V. I. Ritus. *Sov. Phys. JETP.* **29,** 532, (1969).

[3] I. M. Ternov *et.al. Sov. Phys. JETP Letters*. **37**, 288, (1983).

[4] I. M. Ternov, V. N. Rodionov, O. F. Dorofeev. *Sov. Phys. JETP.* **57**, 710, (1983).

[5] A. I. Nikishov, V. I. Ritus. *Sov. Phys. JETP.* **58,** 14, (1983).

[6] E. A. Akhmedov, *Sov. Phys. JETP* **58**, 91, (1983).

[7] W. Becker, R. R. Schlicher, and M. O. Scully, *Nucl. Phys. A* **426**, 125, (1984).

[8] E. A. Akhmedov, *Sov. Phys. JETP* **60,** 884, (1984).

[9] A. Ray, P. Das, S. K. Saha, *et al., Phys. Rev. C* **66,** 012501(R), (2002).

[10] M. Kusaba, M. Imamura, K.Yagi, *et al*., Ann. Report, Jan.Dec. 1989 (Inst. Nucl. Study Univ. Tokyo, Tokyo,1990), p. 57.

[11] G. N. Emery, *Ann. Rev. Nucl. Sci.* **22**, 165, (1972).

[12] L. N. Ivanov, V. S. Letokhov. *Sov. Phys. JETP.* **41,** 877, (1975).

[13] L. N. Ivanov, V. S. Letokhov. *Sov. Phys. JETP.* **66,** 227, (1987).

[14] B. Pontecorvo, D. H. W. Kirkwood, and G. C. Hanna, *Phys. Rev. Lett.* **75** 982 (1949); W. M. Hubbard, *Phys. Rev. B* **137**, 245, (1965).

[15] Table of Isotopes. The 8th CD Update, Ed. by R. B. Firestone, S. Y. F. Chu, and M. Baglin (Lawrence Berkeley National Laboratory, Univ. California, 1999).

[16] J.Wing, C. M. Stevens, and J. R. Huizenga, *Phys. Rev.* **111**, 590, (1958).

[17] P. K. Hopke, *Phys. Rev. C* **3**, 606, (1971).

[18] B. S. Dzhelepov, L. N. Zyryanova, Yu. P. Suslov. Beta Protsessy. Funktsii dlya Analiza Beta-Spektrov i Elektronnogo Zakhvata (Beta Processes. Functions for the Analysis of Beta-Spectra and Electron Capture (M.: Nauka, 1972)[in Russian].

[19] C. Zener, *Phys. Rev.* **36**, 51, (1930); J. C. Slater, *Phys. Rev.* **36**, 57, (1930).

[20] L. D. Landau and E. M. Lifshitz, Course of Theoretical Physics, Vol. 3: Quantum Mechanics: Non Relativistic Theory (Nauka, Moscow, 1989, 4th ed.; Pergamon, New York, 1977, 3rd ed.); A.S.Davydov, Quantum Mechanics (Nauka, Moscow, 1973; Pergamon, Oxford, 1976).

[21] A. H. Wapstra, G. J. Nijgh, and R. van Lieshout, Nuclear Spectroscopy Tables (North Holland, Amsterdam, 1959; Atomizdat, Moscow, 1960).

[22] I. S. Grigor'ev, E. Z. Meilikhov (Red.) Fizicheskie Velichiny (Physical Values) (M.: Energoatomizdat, 1991)[in Russian].

[23] H. Bethe and E. Salpeter, Quantum Mechanics of One and Two Electron Atoms (Springer, Berlin, 1957).

[24] P. V. Elyutin and V. D. Krivchenkov, Quantum Mechanics with Problems (Fizmatlit, Moscow, 2001) [in Russian].

[25] M.Yu.Romanovskii. JETP Letters. **94**, 425, (2011) [Pis'ma v ZETF. **94** 460 (2011))].

[26] D. T. Watt and R. N. Glover. Phil. Mag. **7**, 105, (1962).

[27] A. Alessandrello, C. Arnaboldi, C. Brofferio, *et al*., *Phys. Rev.* **C 67**, 014323, (2003).

[28] M. Yu. Romanovsky and W. Ebeling. Contrib. *Plasma Phys.* **46**, 195, (2006).

[29] J. Holtsmark. *Ann. Phys.* **58,** 577, (1919).

[30] R. Mantegna and H. E. Stanley. *Phys. Rev. Lett.* **73,** 2946, (1994).

[31] R. G. Winter. *Phys. Rev.* **100**, 142, (1955).

[32] Z. Sujkowski, S. Wycech. *Phys. Rev. C* **70**, 052501(R), (2004).

[33] A. S. Barabash *et al. Nucl. Phys. A* **785**, 371, (2007).

[34] J. D. Vergados. *Phys. Rev. C* **84**, 044328, (2011).

[35] S. Eliseev *et al. Phys. Rev. C* **83**, 038501, (2011).

[36] S. Eliseev *et al*. *Phys. Rev. C* **84**, 012501(R), (2011).

[37] M. Goncharov *et al*. *Phys. Rev. C* **84**, 028501, (2011).

[38] J. Bernabeu, A. De Rujula, C. Jarlskog. *Nucl. Phys. B* **223**, 15, (1983).

[39] V. S. Kolhinen, V. V. Elomaa, T. Eronen, J. Hakala, A. Jokinen, M. Kortelainen, J. Suhonen, J. Äystö. Physics Letters **B 684,** 17, (2010).

[40] D. L. Matthews *et al*. *Phys. Rev. Lett*. **54**, 110, (1985).

[41] B. J. MacGowan. *J. Appl. Phys.* **61**, 5243, (1987).

[42] M.Yu.Romanovskii. Physics-Uspekhi, **55**(7), (2012) [Uspekhi Fizicheskih Nauk.**182**781 (2012)].

[43] V. V. Korobkin, M. Yu. Romanovsky. *Phys. Rev. E* **49**, 2316, (1994).

[44] V. V. Korobkin, M. Yu. Romanovsky. *Laser Part. Beams* **16**, 235, (1998).

[45] P. A. Bagryansky *et al. Fusion Sci. Technol*. **59,** 31, (2011).

[46] N.Tragin *et al*. *Phys. Rev. A* **39,** 2085, (1989).