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Binding Energies of Hydrogenlike Carbon under Maxwellian Dusty Plasma Environment

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ABSTRACT: The analytic form of the electrostatic potential felt by a slowly moving test charge in Maxwellian dusty plasma is developed. It has been shown that the electrostatic potential is composed of three parts: i) the Debye-Hückel screening term, ii) the near-field wake potential and iii) the dust charge perturbation effect. The last two terms depend on the velocity of the test charge, the number density of the plasma electrons and the dust grain parameters. Precise energy eigenvalues of hydrogen-like carbon ion under such plasma environment has been estimated by employing Rayleigh-Ritz variational calculation. The form of the potential facilitates the removal of l-degeneracy and $|m|$-degeneracy in the energy levels. A detailed analysis shows that the energy levels gradually move to the continuum with increasing plasma electron density and the variation of ion velocity. Incidental degeneracy of the energy levels and level crossing phenomena have been observed with the variation of plasma electron density.

Keywords: Dusty plasma, variational method, one-electron atom

I. INTRODUCTION

In recent years, dusty plasmas are attracting considerable attention in the field of plasma physics research. In addition to electrons, ions, neutrals as present in ordinary plasmas, dusty plasmas contain massive particles of nanometer to micrometer size. The dust grains may be metallic, conducting, or made of ice particulates. Plasma with dust particles or grains can be termed as either ‘dust in plasma’ or ‘dusty plasma’ depending on the relative values of three characteristic lengths : i) the dust grain radius (r_d), ii) the average inter-grain distance (a) and iii) the Debye radius (λ_D). For $r_d < \lambda_D < a$, charged dust particles are considered as a collection of isolated screened grains, which corresponds to ‘dust in plasma’. For the condition $r_d < a < \lambda_D$ dust particles participate in the collective behavior and in that case the plasma is said to be ‘dusty plasma’. Dusty plasmas are most abundant in astrophysical objects like in the planetary rings, in cometary tails or in interstellar clouds [1, 2]. Dusty plasmas are also formed in laboratory based experiments like dc and rf- discharges, plasma processing reactors, fusion plasma devices, solid-fuel combustion products *etc.* [3]. Dusty plasmas play important role in formation of plasma crystals as under some plasma conditions dust grains can order themselves into crystal-like structure [4, 5].

There are a number of theoretical studies of plasma wave modulation, transport phenomena of the particles, ion drag forces, phase transitions, crystallization of dust grains under dusty plasma environment [6-13]. But the effect of dusty plasma on the structural properties of atoms is rather scanty [14]. The most important part of such studies is to develop an appropriate model interatomic potential from a pure electrostatic view which can mimic the conditions of such plasma environment. Unlike the plasma modeled by exponentially screened Coulomb potential, the model potential for dusty plasma contains a complex character [14-18]. The closed form of the far-field potential felt by a slowly moving test charge through unmagnetized dusty plasma in the spherical polar co-ordinate was first derived by Shukla [15]. It is shown that the effective potential consists of three parts: exponentially screened Coulomb part, far-field wake potential part and dust charge fluctuation term. Shukla *et al.* [16] developed another form of the far-field potential of a slowly moving test charge in a plasma that consisted of positive dust grains and electrons. The dust grain charge fluctuations and collisions among neutral atoms, electrons and dust grains were taken into account. In the work of Moslem *et al.* [17], the Debye–Hückel screening potential and oscillatory wake field potential distribution around a test charge particle moving in the dusty plasma medium were derived by solving the linearized Vlasov equation along with the Poisson equation. Ali *et al.* [18] also used Vlasov-Poisson equation to formulate the electrostatic potential caused by a test charge in unmagnetized non-Maxwellian dusty plasma where the plasma particles are : superthermal hot electrons, cold fluid electrons, neutralizing cold cations and charge fluctuating isolated dust grains.

The aim of the present paper is to formulate the near field potential felt by an atom/ion moving slowly through unmagnetized dusty plasma and apply the potential to find the binding energies of one-electron ion. The binding energies of moving hydrogen-like carbon (C^{5+}) ion under different conditions of the classical dusty plasma are estimated by using variational method. It is observed that the *l*-degeneracy of the hydrogenic energy levels corresponding to a principal quantum number is lifted under this potential. Moreover, a partial removal of the *m*-degeneracy is also observed. In particular, we have calculated the energy values of C^{5+} ion in $1s_0$, $2s_0$, $2p_0$ and $2p_1$ states by varying the velocity of the ion and the plasma electron density as well. The details of the formulation of the inter-atomic potential for slowly moving test charge under dusty plasma is given in the Sec. II, the details of the variational method used for the atomic structure calculation is given in Sec. III, computational results are given in Sec. IV and final conclusion in Sec. V.

II. NEAR-FIELD POTENTIAL FELT BY A SLOWLY MOVING “TEST CHARGE” IN CLASSICAL DUSTY PLASMA

The field ($\vec{D}$) of a charge q moving with a velocity $\vec{u}$ in a dielectric medium is given by the Poisson’s equation [20],

$$\vec{\nabla}.\vec{D} = \frac{q}{\varepsilon_0}\delta(\vec{r} - \vec{u}t) \tag{1}$$

where $\vec{D} = -D\vec{\nabla}\varphi$, D being the dielectric constant of the medium and φ being the potential in the medium. Using the relation and making a Fourier transform followed by an inverse Fourier transform, we obtain the expression for the potential as [20]

$$\varphi(\vec{r}) = \frac{q}{8\pi^3\varepsilon_0}\int \frac{e^{j\vec{k}.\vec{r}}}{k^2 D\left(\vec{k}, -\vec{k}\cdot\vec{u}\right)} d^3\vec{k} \tag{2}$$

Here $j = \sqrt{-1}$ and ε_0 is the permittivity of free space. The dielectric constant of the medium is given by,

$$D\left(\vec{k}, -\vec{k}\cdot\vec{u}\right) = 1 + \chi_e + \chi_i + \chi_d \tag{3}$$

where, $\chi_s \big|_{s=e,i,d}$ is the electric susceptibility for the plasma species ‘*s*’ (*s*=*e,i,d* corresponding to electron, ion and dust, respectively).

Considering the Maxwell-Boltzmann distribution for the plasma particles, the electric susceptibility due to the thermal motion of plasma electrons and ions is given by [20],

$$\chi_s\big|_{s=e,i} = \frac{1}{k^2\lambda_s^2}\left(1 - j\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{kv_{ts}}\right) \tag{4}$$

The electric susceptibility due to dust grain charging and thermal motion is given as [15],

$$\chi_d = \frac{1}{k^2\lambda_d^2}\left(1 - j\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{kv_{td}}\right) + \frac{1}{k^2\lambda_i^2}\frac{\nu_{ed}}{\nu_c + j\vec{k}.\vec{u}}$$

$\lambda_s = \frac{\varepsilon_0 m_s T_s}{n_s q_s^2}$ is the Debye screening length of the plasma species 's'. T_s, m_s, q_s and n_s are absolute temperature, mass, charge and equilibrium number density respectively of species 's'. The thermal speed of the plasma species 's' is given by, $v_{ts} = \sqrt{\frac{K_B T_s}{m_s}}$ where K_B is the Boltzmann constant.

If we consider that the dust grain contains negative charges only, then the quasi-charge neutrality condition within the effective Debye-sphere of the plasma becomes, $n_i q_i = n_e q_e + n_d q_d$. For $\nu_c >> \left|\vec{k}\cdot\vec{u}\right|$, electric susceptibility due to the dust grain becomes,

$$\chi_d \approx \frac{1}{k^2\lambda_d^2}\left(1 - j\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{kv_{td}}\right) + \frac{\nu_{ed}}{k^2\lambda_i^2\nu_c} - j\frac{\nu_{ed}\left(\vec{k}.\vec{u}\right)}{k^2\lambda_i^2\nu_c^2} \tag{5}$$

Using (4) and (5), one can obtain the modified form of (3) as,

$$D\left(\vec{k}, -\vec{k}\cdot\vec{u}\right) = 1 + \frac{1}{k^2}\left(\sum_{s=e,i,d}\frac{1}{\lambda_s^2} + \frac{\nu_{ed}}{\nu_c\lambda_i^2}\right) - j\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{k^3}\sum_{s=e,i,d}\frac{1}{v_{ts}\lambda_s^2} - j\frac{\nu_{ed}\left(\vec{k}.\vec{u}\right)}{k^2\lambda_i^2\nu_c^2}$$

$$= 1 + \frac{1}{k^2\lambda_t^2} - j\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{k^3}\sum_{s=e,i,d}\frac{1}{v_{ts}\lambda_s^2} - j\frac{1}{k^2\lambda_i^2}\frac{\nu_{ed}\left(\vec{k}.\vec{u}\right)}{\nu_c^2}$$

$$= \frac{1+k^2\lambda_t^2}{k^2\lambda_t^2}\left[1 - j\frac{k^2\lambda_t^2}{1+k^2\lambda_t^2}\left\{\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{k^3}\sum_{s=e,i,d}\frac{1}{v_{ts}\lambda_s^2} + \frac{1}{k^2\lambda_i^2}\frac{\nu_{ed}\left(\vec{k}.\vec{u}\right)}{\nu_c^2}\right\}\right] \tag{6}$$

where

$$\frac{1}{\lambda_t^2} = \sum_{s=e,i,d}\frac{1}{\lambda_s^2} + \frac{\nu_{ed}}{\nu_c\lambda_i^2} \tag{7}$$

For very slowly moving atom/ion i.e. $v_{ts} >> u$, which means the thermal Mach number (defined as the ratio of ion velocity and thermal velocity of plasma particles) remains below unity, the inverse of dielectric function becomes,

$$\frac{1}{D\left(\vec{k}, -\vec{k}\cdot\vec{u}\right)} \approx \frac{k^2\lambda_t^2}{1+k^2\lambda_t^2}\left[1 + j\frac{k^2\lambda_t^2}{1+k^2\lambda_t^2}\left\{\sqrt{\frac{\pi}{2}}\frac{\vec{k}.\vec{u}}{k^3}\sum_{s=e,i,d}\frac{1}{v_{ts}\lambda_s^2} + \frac{1}{k^2\lambda_i^2}\frac{\nu_e\left(\vec{k}.\vec{u}\right)}{\nu_c^2}\right\}\right]$$

$$= \frac{k^2\lambda_t^2}{1+k^2\lambda_t^2} + j\sigma_1\frac{k^2}{\left(1+k^2\lambda_t^2\right)^2}\cos\left(\eta+\theta\right) + j\sigma_2\frac{k^3}{\left(1+k^2\lambda_t^2\right)^2}\cos\left(\eta+\theta\right) \tag{8}$$

where, $$\sigma_1 = \sqrt{\frac{\pi}{2}} u \lambda_t^4 \sum_{s=e,i,d} \frac{1}{v_{ts}\lambda_s^2} \text{ and } \sigma_2 = \frac{u v_{ed} \lambda_t^4}{\lambda_i^2 v_c^2} \tag{9}$$

Here, θ is the angle between $\vec{r}$ and $\vec{u}$ and $(\theta + \eta)$ is the angle between $\vec{k}$ and $\vec{u}$. Using equation (8) one can dissolve equation (2) into three parts that may be given as follows:

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3$$

where we define the first part as

$$\varphi_1 = \frac{q}{8\pi^3 \varepsilon_0} \int \frac{\lambda_t^2}{1 + k^2 \lambda_t^2} e^{i\vec{k}.\vec{r}} d^3\vec{k} = \frac{q}{4\pi\varepsilon_0 r} e^{-r/\lambda_t} \tag{10}$$

The method of solving the integral can be found in ref. [21]. Thus the potential φ_1 is of the form of Debye-Hückel potential [22], where λ_t signifies the effective or total *Debye length* of the plasma. The inverse of *Debye length* is known as *Debye parameter* or simply *screening parameter* (μ) *i.e.* $\mu = 1/\lambda_t$. The second part of the potential φ is

$$\varphi_2 = j \frac{q\sigma_1}{8\pi^3 \varepsilon_0} \int \frac{k^2 e^{jkr\cos\eta}}{(1 + k^2 \lambda_t^2)^2} \cos(\eta + \theta) \sin\eta d\eta d\tau dk \tag{11}$$

Here we have used the volume element $d^3\vec{k} = k^2 \sin\eta d\eta d\tau dk$ in spherical polar coordinate (k, η, τ). One can get the imaginary solution of the angular part of the integral (11) as,

$$\int_{\tau=0}^{2\pi} \int_{\eta=0}^{\pi} e^{jkr\cos\eta} \cos(\eta + \theta) \sin\eta d\eta d\tau = \frac{4\pi\cos\theta}{j} \left(\frac{\cos kr}{kr} - \frac{\sin kr}{k^2 r^2} \right) \tag{12}$$

Now using (12), equation (11) reduces to

$$\varphi_2 = -\frac{q\sigma_1 \cos\theta}{2\pi^2 \varepsilon_0} \int_{k=0}^{\infty} \frac{k^2 j_1(kr)}{(1 + k^2 \lambda_t^2)^2} dk \tag{13}$$

Here $j_1(kr)$ is the spherical Bessel function of first order [19]. The solution of integral (13) can be done using the standard Meijer's G function [19] and the solution technique described in [21], where in the limit $r < 2\lambda_t$ the above integral becomes

$$\int_{k=0}^{\infty} \frac{k^2 j_1(kr)}{(1 + k^2 \lambda_t^2)^2} dk = \frac{1}{2\lambda_t^4} r K_0\left(r/\lambda_t \right)$$, $K_0(x)$ being the Macdonald function or modified Bessel function of second kind [19].

Using the above result and putting σ_1 from equation (9) one can get from equation (13) as,

$$\varphi_2 = -CrK_0\left(r/\lambda_t \right) \cos\theta \tag{14}$$

This potential is called the *near field wake potential* and $C = \frac{qu}{4\pi\varepsilon_0\sqrt{2\pi}} \sum_{s=e,i,d} \frac{1}{v_{ts}\lambda_s^2}$ is the wake-coefficient.

Let us now consider the third part of potential,

$$\varphi_3 = j\frac{q\sigma_2}{8\pi^3\varepsilon_0}\int \frac{k^3 e^{jkr\cos\eta}}{\left(1+k^2\lambda_t^2\right)^2}\cos(\eta+\theta)\sin\eta d\eta d\tau dk \tag{15}$$

Here, the angular part of the integration is same as the angular part in wake potential. So, by using equation (12) one can rearrange equation (15) in the following way,

$$\varphi_3 = \frac{q\sigma_2\cos\theta}{2\pi^2\varepsilon_0 r^2}\int_{k=0}^{\infty}\frac{k\left(kr\cos kr - \sin kr\right)}{\left(1+k^2\lambda_t^2\right)^2}dk \tag{16}$$

Let us now consider a standard integral [23],

$$\int_{k=0}^{\infty}\frac{k\sin kr}{\left(1+k^2\lambda_t^2\right)^2}dk = \frac{\pi r}{4\lambda_t^3}e^{-r/\lambda_t} \tag{17}$$

$$\therefore \int_{k=0}^{\infty}\frac{k^2\cos kr}{\left(1+k^2\lambda_t^2\right)^2}dk = \frac{\partial}{\partial r}\int_{k=0}^{\infty}\frac{k\sin kr}{\left(1+k^2\lambda_t^2\right)^2}dk = \frac{\pi}{4\lambda_t^3}\left(1-\frac{r}{\lambda_t}\right)e^{-r/\lambda_t} \tag{18}$$

Using the integrals (17) and (18) and the value of σ_2 from equation (9), equation (16) takes the following form,

$$\varphi_3 = -De^{-r/\lambda_t}\cos\theta \tag{19}$$

where, $D = \frac{q}{4\pi\varepsilon_0}\frac{uv_{ed}}{2\lambda_i^2 v_c^2}$. This potential is due to dust perturbation part and will vanish if the moving test charge is static *i.e.* $u = 0$ and/or the electron-dust collision is absent i.e. $v_{ed} = 0$.

III. CALCULATION OF ENERGY LEVELS OF HYDROGENLIKE ION

To estimate the modified non-relativistic energy eigenvalues of slowly moving hydrogen-like ion in the presence of an external classical dusty plasma environment, Rayleigh-Ritz variation calculation has been done (a.u. is used hereafter). The expectation value of kinetic energy is given by,

$$\langle T\rangle = \int\left[\left(\frac{\partial\Psi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\Psi}{\partial\theta}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial\Psi}{\partial\omega}\right)^2\right]d^3\vec{r} \tag{20}$$

Where $d^3\vec{r} = r^2\sin\theta d\theta d\omega dr$ is the volume element in spherical polar (r,θ,ω) co-ordinate ($0\le r\le\infty$, $0\le\theta\le\pi$, $0\le\omega\le 2\pi$). The effective potential energy of the atom can be written as,

$$V_{eff} = -\frac{q}{r}e^{-r/\lambda_t} + CrK_0\left(r/\lambda_t\right)\cos\theta + De^{-r/\lambda_t}\cos\theta \tag{21}$$

Thus the expectation value of potential energy is given by,

$$\langle V \rangle = \int V_{eff} \Psi^2 d^3\vec{r} \tag{22}$$

The normalization term is given by,

$$\langle N \rangle = \int \Psi^2 d^3\vec{r} \tag{23}$$

The trial wavefunction is taken as, $\Psi = R_{nl}(\alpha r) Y_{lm}(\theta, \omega) A(\theta)$, where $R_{nl}(\alpha r)$ is the radial part of hydrogenic wavefunction [24] with 'α' as variation parameter, $Y_{lm}(\theta, \omega)$ is the spherical harmonics [24] and $A(\theta) = (\gamma + \beta\cos\theta)$ is the orbital distortion part [21] with 'γ' and 'β' as variation parameters. For the static ion (u = 0), wake and dusty potentials will be absent and in this case we set γ = 1 and β = 0.

Let us now consider the one-electron auxiliary integral for radial part,

$$W_1(p, \rho) = \int_{r=0}^{\infty} r^p e^{-\rho r} dr = \frac{p!}{\rho^{p+1}} \tag{24}$$

and the integral necessary to evaluate the expectation value of wake potential,

$$U_1(p, v, \rho, \xi) = \int_{r=0}^{\infty} r^{p-1} e^{-\rho r} K_v(\xi r) dr$$

$$= \frac{\sqrt{\pi}\,(2\xi)^v}{(\rho + \xi)^{p+v}} \frac{\Gamma(p+v)\Gamma(p-v)}{\Gamma\left(p + \frac{1}{2}\right)} {}_2F_1\left(p + v, v + \tfrac{1}{2}; p + \tfrac{1}{2}; \frac{\rho - \xi}{\rho + \xi}\right) \tag{25}$$

where, ${}_2F_1(a, b; c; x)$ is the confluent Hypergeometric function and $\mathrm{Re}(p) > |\mathrm{Re}(v)|$ and $\mathrm{Re}(\rho + \xi) > 0$ [23]. The expectation values using (20),(22) and (23) for the $1s_0$, $2s_0$, $2p_0$ and $2p_1$ states are given as follows.

$1s_0$ – state

Using the trial wavefunction as $\Psi(r, \theta, \omega) = e^{-\alpha r} A(\theta)$, the expectation values can be derived as

$$\langle T \rangle = \left(\gamma^2 + \frac{\beta^2}{3}\right)\alpha^2 W_1(2, 2\alpha) + \frac{2\beta^2}{3} W_1(0, 2\alpha)$$

$$\langle V \rangle = -2q\left(\gamma^2 + \frac{\beta^2}{3}\right) W_1(1, 2\alpha + \mu) + \frac{4}{3} C\gamma\beta U_1(4, 0, 2\alpha, \mu) + \frac{4}{3} D\gamma\beta W_1(2, 2\alpha + \mu)$$

$$\langle N \rangle = 2\left(\gamma^2 + \frac{\beta^2}{3}\right) W_1(2, 2\alpha)$$

$2s_0$ – state

Using the trial wavefunction as $\Psi(r, \theta, \omega) = (1 - \alpha r) e^{-\alpha r} A(\theta)$, the expectation values can be derived as

$$\langle T\rangle=\alpha^2\left(\gamma^2+\frac{\beta^2}{3}\right)\left[4W_1(2,2\alpha)-4\alpha W_1(3,2\alpha)+\alpha^2W_1(4,2\alpha)\right]+\frac{2\beta^2}{3}\left[W_1(0,2\alpha)-2\alpha W_1(1,2\alpha)+\alpha^2W_1(2,2\alpha)\right]$$

$$\langle V\rangle=-2q\left(\gamma^2+\frac{\beta^2}{3}\right)\left[W_1(1,2\alpha+\mu)-2\alpha W_1(2,2\alpha+\mu)+\alpha^2W_1(3,2\alpha+\mu)\right]$$

$$+\frac{4}{3}C\gamma\beta\left[U_1(4,0,2\alpha,\mu)-2\alpha U_1(5,0,2\alpha,\mu)+\alpha^2U_1(6,0,2\alpha,\mu)\right]$$

$$+\frac{4}{3}D\gamma\beta\left[W_1(2,2\alpha+\mu)-2\alpha W_1(3,2\alpha+\mu)+\alpha^2W_1(4,2\alpha+\mu)\right]$$

$$\langle N\rangle=2\left(\gamma^2+\frac{\beta^2}{3}\right)\left[W_1(2,2\alpha)-2\alpha W_1(3,2\alpha)+\alpha^2W_1(4,2\alpha)\right]$$

$2p_0$ – state

Using the trial wavefunction as, $\Psi(r,\theta,\omega)=re^{-\alpha r}\cos\theta A(\theta)$, the expectation values can be derived as

$$\langle T\rangle=\left(\frac{\gamma^2}{3}+\frac{\beta^2}{5}\right)\left[W_1(2,2\alpha)-2\alpha W_1(3,2\alpha)+\alpha^2W_1(4,2\alpha)\right]+2\left(\frac{\gamma^2}{3}+\frac{4\beta^2}{15}\right)W_1(2,2\alpha)$$

$$\langle V\rangle=-2q\left(\frac{\gamma^2}{3}+\frac{\beta^2}{5}\right)W_1(3,2\alpha+\mu)+\frac{4}{5}C\gamma\beta U_1(6,0,2\alpha,\mu)+\frac{4}{5}D\gamma\beta W_1(4,2\alpha+\mu)$$

$$\langle N\rangle=2\left(\frac{\gamma^2}{3}+\frac{\beta^2}{5}\right)W_1(4,2\alpha)$$

$2p_1$ – state

Using the trial wavefunction as, $\Psi(r,\theta,\omega)=-re^{-\alpha r}\sin\theta e^{j\omega}A(\theta)$ the expectation values can be derived as

$$\langle T\rangle=2\left(\frac{\gamma^2}{3}+\frac{\beta^2}{15}\right)\left[W_1(2,2\alpha)-2\alpha W_1(3,2\alpha)+\alpha^2W_1(4,2\alpha)\right]+4\left(\frac{\gamma^2}{3}+\frac{\beta^2}{5}\right)W_1(2,2\alpha)$$

$$\langle V\rangle=-4q\left(\frac{\gamma^2}{3}+\frac{\beta^2}{15}\right)W_1(3,2\alpha+\mu)+\frac{8}{15}C\gamma\beta U_1(6,0,2\alpha,\mu)+\frac{8}{15}D\gamma\beta W_1(4,2\alpha+\mu)$$

$$\langle N\rangle=4\left(\frac{\gamma^2}{3}+\frac{\beta^2}{15}\right)W_1(4,2\alpha)$$

The variational energy eigenvalue is now given as,

$$E=\frac{\langle T\rangle+\langle V\rangle}{\langle N\rangle}=E_{nlm}(\alpha,\beta,\gamma) \tag{26}$$

The parameters (α,β,γ) have been optimized using Nelder-Mead algorithm [25].

IV. RESULTS AND DISCUSSION

The energy values of $1s_0$, $2s_0$, $2p_0$ and $2p_1$ states of C^{5+} ion are given in the table 1 where, the electron densities (n_e) are taken in such a way that the dust radius (r_d) remains smaller than the effective Debye length (λ_t) and different ion velocities are considered for which the thermal Mach number remains below unity. We have chosen typical size of dust radius as $r_d = 0.5\ nm$, charge accumulated on dust grain $q_d = 100q_e$ and mass of dust grains as $m_d = 12000m_H$, where m_H is the mass of hydrogen atom.

In the table 1, the first row corresponding to each state indicates the energy eigenvalue of the free static C^{5+} ion. For a fixed value of ion velocity (u) the energy eigenvalues for all the states decreases as n_e increases. Similar feature can be seen as the ion velocity (u) increases for a fixed electron density n_e. But the amount of decrease of energy in the former case is much greater than the later one. Thus the effect of static screening or Debye-Hückel part *i. e.* first part of the effective potential (21), which is a function of plasma electron density (n_e) and dust parameters, is more pronounced than the effect of the second and the third part of the effective potential (21) namely, wake-part and dusty-part, where the later two parts are dependent on ion velocity u and plasma electron density (n_e).

As shown in the table, for the static case (u = 0) due to the effect of Debye-Hückel part in the potential, the *l*-degeneracy gets removed at each density and as a result the energies of $2s_0$, $2p_0$ and $2p_1$ states become different.

Table 1
The energy eigenvalues –*E* (a.u.) of $1s_0$, $2s_0$, $2p_0$ and $2p_1$ states of C^{5+} ion moving in dusty plasma estimated with different sets of electron number density (n_e in m^{-3}) and ion velocity (u in ms^{-1})

State	n_e (m^{-3})	u (ms^{-1})	-*E* (a.u.)	*State*	n_e (m^{-3})	u (ms^{-1})	-*E* (a.u.)
	-	-	18.0		-	-	4.5
		0	17.98216023			0	4.482196542
		100	17.98216023			100	4.482162312
		500	17.98216022		10^{20}	500	4.482161888
	10^{20}	1000	17.98216020			1000	4.482161358
		5000	17.98216008			5000	4.482157116
I_{s0}		10000	17.98215992	$2p_0$		10000	4.482151815
		0	17.44281241			0	3.957701449
		100	17.44281012			100	3.957652599
	10^{23}	500	17.44280094		10^{23}	500	3.957476414
		1000	17.44278948			1000	3.957276511
		5000	17.44271391			5000	3.955684605
		10000	17.44263049			10000	3.953694721
	-	-	4.5		-	-	4.5
		0	4.482198883			0	4.482196542
		100	4.482175960			100	4.482137215
	10^{20}	500	4.482175932		10^{20}	500	4.482137073
		1000	4.482175898			1000	4.482136897
		5000	4.482175675			5000	4.482135483
$2s_0$		10000	4.482175602	$2p_1$		10000	4.482133716
		0	3.961873870			0	3.957701449
		100	3.961868049			100	3.957656795
		500	3.961844765			500	3.957598066
	10^{23}	1000	3.961815660		10^{23}	1000	3.957524654
		5000	3.961664635			5000	3.956990871
		10000	3.961452745			10000	3.956327567

Because of the *cos* θ term in the near field wake part and the dusty part in the effective potential, the degeneracy of energy eigenvalues with respect to the absolute value of magnetic quantum number (|*m*|) is lifted (corresponding to given *n* and *l*). For example, from the table 1, it can be seen that for ion velocity u = 1000 m/s and plasma electron density $n_e = 10^{23}\ m^{-3}$ the energy eigenvalues of $2p_0$ and $2p_1$ states are -3.95727651 a.u. and -3.95752465 a.u., which indicates that both the states are no longer degenerate. It is also noteworthy that for $n_e=10^{20}\ m^{-3}$ the $2p_0$ state energetically lies below $2p_1$, while for $n_e=10^{23}\ m^{-3}$, the $2p_0$ state energetically moves above to the $2p_1$ state, giving rise to the level-crossing phenomenon. Thus one may opine that the relative positions of the states corresponding to same *n* and *l*-values and different |*m*| values depend on the plasma density of the dusty plasma environment. Moreover, it can also be argued that two different levels can be made degenerate *i.e.* incidental degeneracy [21] may occur by tuning the plasma parameters.

If the dust charge perturbation term is removed from the effective potential (by setting v_{ed}= 0), the energy of $1s_0$ state for $n_e = 10^{23}\ m^{-3}$ and u = 1000 ms^{-1}, becomes -17.44377726 a.u., whereas with dust potential under the same plasma conditions and ion velocity, the energy of $1s_0$ state becomes -17.44278948 a.u. (as shown in table 1). Thus in the presence of dust potential part, the energy of $1s_0$ state becomes more positive by an amount of 9.8778×10^{-4} a.u.

V. CONCLUSION

The electrostatic potential for a moving ion under classical dusty plasma is derived where the thermal Mach number remains less than unity and dust grain radius is smaller than the effective screening length of the plasma. Subsequently, the effect of such potential on the change of the energy eigenvalues of different states of hydrogen-like carbon ion is studied under the framework of Rayleigh-Ritz variational method. The removal of accidental (*l*) degeneracy and absolute magnetic quantum number (|*m*|) degeneracy are reported in case of an ion moving in the dusty plasma environment. Level-crossing phenomenon has been observed between $2p_0$ and $2p_1$ states with the variation of plasma electron density. The present form of the potential may be useful for calculating spectral properties of other ions within dusty plasma surrounding. The energy eigenvalues reported here may serve as benchmark for future theoretical research and also for experimental measurements under such plasma environment.

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