IRAMP
7(1), June 2016, pp. 21-29

ISSN: 2229-3159

Generalization of Analytical Results for Lorentz-Doppler Profiles of Hydrogen/Deuterium Lines

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ABSTRACT: We extend the analytical results for the Lorentz-Doppler profiles of highly-excited hydrogen/deuterium spectral lines for the arbitrary strength of the magnetic field *B*, obtained in paper by one of us (J. Quant. Spectr. Rad. Transfer 156 (2015) 24) for observations parallel or perpendicular with respect *B*, to arbitrary angles of the observation ψ. We study the evolution of the effect of the suppression of the π- components (compared to the σ-components) as the angle of the observation ψ decreases from 90 degrees, *i.e.*, as ψ decreases from the value, at which this counterintuitive effect was discovered in the paper quoted above. We show that the suppression of the π-components (compared to the σ-components) occurs at the perpendicular or close to the perpendicular direction of observation (with respect to the magnetic field *B*), but disappears at lower values of the angle of the observation. For observations perpendicular to *B*, the halfwidth of the Lorentz-Doppler profiles is a non-monotonic function of the magnetic field, which is a counterintuitive result. Our findings should be important, *e.g.*, for spectroscopic diagnostics of edge plasmas of various magnetic fusion devices around the world.

1. INTRODUCTION

Strongly-magnetized plasmas are encountered both in astrophysics (*e.g.*, in Sun spots, in the vicinity of white dwarfs etc.) and in laboratory plasmas (*e.g.*, in magnetic fusion devices). In such plasmas, as hydrogen/deuterium atoms move across the magnetic field *B* with the velocity *v*, they experience a Lorentz electric field $E_L = v \times B/c$ in addition to other electric fields. The Lorenz field has a distribution because the atomic velocity *v* has a distribution. So, for radiating hydrogen/deuterium atoms this becomes an additional source of the *broadening* of spectral lines.

In paper [1] were described situations where the Lorentz broadening serves as the primary broadening mechanism of Highly-excited Hydrogen/deuterium Spectral Lines (HHSL). One example discussed in paper [1] was HHSL emitted from edge plasmas of tokamaks. In laboratory plasmas, HHSL are used for measuring the electron density at the edge plasmas of tokamaks (see, *e.g.*, papers [2, 3] and Sect. 4.3 of review [4]) and in radiofrequency discharges (see, *e.g.*, paper [5] and book [6]).

Another example discussed in paper [1] was HHSL emitted from the solar chromosphere. They are observed and used for measuring the electron density in the solar chromosphere (see, *e.g.*, paper [7]).

One of the most interesting features of these situations is that the combination of Lorentz and Doppler broadenings cannot be taken into account simply as a convolution of these two broadening mechasnisms, as it was pointed out for the first time in paper [8]. The Lorentz and Doppler broadening intertwine in a more complicated way. Indeed, let us consider a Stark component of HHSL. Its Lorentz-Doppler profile in the frequency scale is proportional (in the laboratory reference frame) to $\delta[\Delta\omega - (\omega_0 v/c)\cos\alpha - (kX_{\alpha\beta}Bv/c)\sin\vartheta]$, where in the argument of this δ-function the quantity α is the angle between the direction of observation and the atomic velocity *v*, and ϑ is the angle between vectors *v* and *B*.

In paper [1] was derived a general expression for the Lorentz-Doppler profiles of HHSL for the arbitrary strength of the magnetic field *B* and for the arbitrary angle of the observation ψ with respect *B*. However, more *specific* analytical results were obtained in paper [1] only for ψ = 0 and ψ = 90 degrees. It was shown that a relatively strong magnetic field causes a significant *suppression of* π*-components* compared to σ-components for the observation at ψ = 90 degrees, which was a counterintuitive result.*/

In the present paper we obtain *specific* analytical results for the Lorentz-Doppler profiles of HHSL for the arbitrary strength of the magnetic field *B* and for an *arbitrary angle of the observation* ψ. In particular, we show that the effect of the suppression of π-components at a relatively strong magnetic field rapidly diminishes as the angle of observation ψ decreases from 90 degrees.

2. ANALYTICAL RESULTS

For an arbitrary angle ψ between the direction of observation and the magnetic field, the relative configuration of the vectors *B*, E_L, and *v*, as well as the choice of the reference frame is shown in Figure 1.

In paper [1] for obtaining universal analytical results the following dimensionless notations were introduced:

$$w = c\Delta\omega/v_T\omega_0 = c\Delta\lambda/v_T\lambda_0,\; b = kX_{\alpha\beta}B/\omega_0,\; u = v/v_T, \tag{1}$$

Here *w* is the scaled detuning from the unperturbed frequency ω_0 or from the unperturbed wavelength λ_0 of a hydrogen spectral line, *B* is the scaled magnetic field, and u is the atomic velocity scaled with respect to the atomic thermal velocity v_T. The quantities *k* and $X_{\alpha\beta}$ in Eq. (1) are

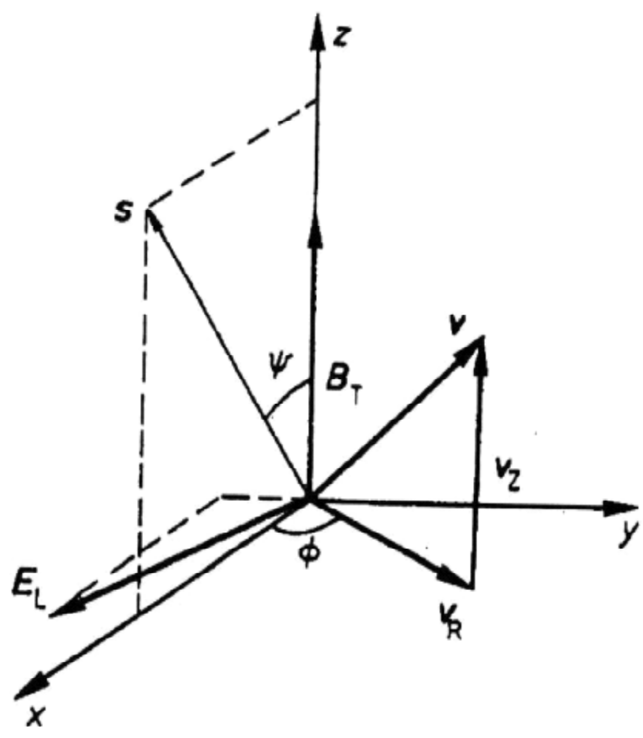

Figure 1: Relative configuration of the magnetic *B* and Lorentz E_L fields and of the direction of the observation *s* ("*s*" stands for "spectrometer"). The *z* axis is along *B*. The direction of the observation *s* constitutes a non-zero angle *y* with *B*. The *xz* plane is spanned on vectors *B* and *s*. The atomic velocity *v* has a component v_z along *B* and a component v_R perpendicular to *B*. The component v_R constitutes an angle φ with the *x* axis.

*/ We note in passing that in paper [1] there were minor typographic errors in Eqs. (31), and (32). In Eq. (31), the factor in front of the integral should be $\pi^{-1}|2w|^{-1/2}$. In Eq. (32), the factor in front of the last brackets should be $[\Gamma(1/4)\Gamma(-1/4)]^{-1}|w|^{-1/2}$. Also, in Figures 9 and 10 from paper [1], the profiles of π-components were not to scale. The corrected Figures 9 and 10 are presented in Appendix.

$$k = 3\hbar/(2m_e e),\ X_{\alpha\beta} = n_\alpha (n_1 - n_2)_\alpha - n_\beta (n_1 - n_2)_\beta, \tag{2}$$

where n_1, n_2 are the parabolic quantum numbers, and n is the principal quantum numbers of the upper (subscript α) and lower (subscript β) Stark sublevels involved in the radiative transition.

A general expression for the Lorentz-Doppler profiles of components of HHSL for the arbitrary strength of the magnetic field B and for the arbitrary angle of the observation ψ with respect B was derived in paper [1] in the form of the following triple integral

$$I(w,b,\psi) = \int_0^\infty du_z\, f_z(u_z) \int_0^\infty du_R\, f_R(u_R) \int_0^\infty (d\phi/\pi) g(\psi,\phi) \delta\left[w - u_z cos\ \psi - u_R\left(b + \sin\sin\psi\ \cos\phi\right)\right], \tag{3}$$

where

$$f_z(u_z) = \frac{1}{\sqrt{\pi}} e^{-u_z{}^2},\ f_R(u_R) = 2u_R e^{-u_R{}^2},\ 0 < \psi < \frac{\pi}{2}, \tag{4}$$

and g(ψ, ϕ) are factors different for π- and σ- components:

$$g_\pi(\psi) = 1 - \sin^2\psi \sin^2\phi,\ g_\sigma(\psi) = \frac{1}{2}(1 + \sin^2\psi \sin^2\phi). \tag{5}$$

We note that the functions $f_z(u_z)$ and $f_R(u_R)$ are, respectively the one-dimensional and the two-dimensional Maxwell distributions of the scaled atomic velocity $u = v/v_T$.

For the particular cases of ψ = 0 and ψ = 90 degrees, the triple integral from Eq. (3) immediately reduces to double integrals (still having the δ-function in the integrand), as given by Eqs. (9) and (21) from paper [1], respectively. Then the properties of the δ-function were used for performing the angular integration in paper [1], leading to the specific analytical results ψ = 0 and ψ = 90 degrees in the form of a single integral.

In the present paper we consider the angle ψ to be arbitrary, so that we have to start from the triple integral given by Eq. (3). In distinction to paper [1], instead of using the δ-function in the integrand for performing the angular integration, we use it for integrating over u_z. The root of the argument of the delta function is given by:

$$u_z = \frac{w - u_r\left(b + \sin\psi\cos\phi\right)}{\cos\psi}. \tag{6}$$

Employing the properties of the δ-function, we get:

$$I(w,b,\psi) = \frac{2}{\pi^{\frac{3}{2}}\cos\psi} \int_0^\infty du_R u_R e^{-u_R{}^2} \int_0^\pi d\phi\, e^{-\frac{[w - u_R(b+\sin\psi\cos\phi)]^2}{\cos^2\psi}} g(\psi,\phi)$$

$$= \frac{2e^{-\frac{w^2}{\cos^2\psi}}}{\pi^{\frac{3}{2}}\cos\psi} \int_0^\infty du_R u_R e^{-u_R{}^2} \int_0^\pi d\phi\, e^{-\frac{u_R{}^2(b+\sin\psi\cos\phi)^2 - 2wu_R(b+\sin\psi\cos\phi)}{\cos^2\psi}} g(\psi,\phi)$$

$$= \frac{2e^{-\frac{w^2}{\cos^2\psi}}}{\pi^{\frac{3}{2}}\cos\psi} \int_0^\pi d\phi \int_0^\infty du_R u_R e^{-u_R{}^2\left[1 + \frac{(b+\sin\psi\cos\phi)^2}{\cos^2\psi}\right] + 2wu_R\frac{b+\sin\psi\cos\phi}{\cos^2\psi}} g(\psi,\phi) \tag{7}$$

Then we perform the integration over u_R to obtain:

$$I(w,b,\psi)=\frac{e^{-\frac{w^2}{\cos^2\psi}}}{\pi^{\frac{3}{2}}\cos\psi}\int_0^{\pi}\frac{g(\psi,\phi)}{\left[a(b,\psi,\phi)\right]^{\frac{3}{2}}}\left\{\sqrt{a(b,\psi,\phi)}+\sqrt{\pi}c(w,b,\psi,\phi)e^{\frac{c(w,b,\psi,\phi)^2}{a(b,\psi,\phi)}}\left[1+Erf\frac{c(w,b,\psi,\phi)}{\sqrt{a(b,\psi,\phi)}}\right]\right\}d\phi \qquad (8)$$

where

$$a(b,\psi,\phi)=1+\frac{(b+\sin\psi\cos\phi)^2}{\cos^2\psi},\ c(w,b,\psi,\phi)=w\frac{b+\sin\psi\cos\phi}{\cos^2\psi}. \qquad (9)$$

Thus, even for the general case of an arbitrary angle of the observation ψ, we managed to perform analytically two integrations and to reduce the result to just a single integral.

Figures 2-4 present Lorentz-Doppler profiles of p-components of HHSL calculated by Eqs. (8), (9). Each figure shows profiles for five values of the angle ψ (in degrees): 0, 20, 45, 70, and 90. We note that, for example, 20 degrees is the actual angle of observation for the spectroscopic diagnostics at the tokamak EAST in China (other tokamaks around the world have different angles of observation). Figures 2-4 differ from each other by the value of the scaled magnetic field b (defined in Eq. (1)): b = 0.2, 1, and 5.

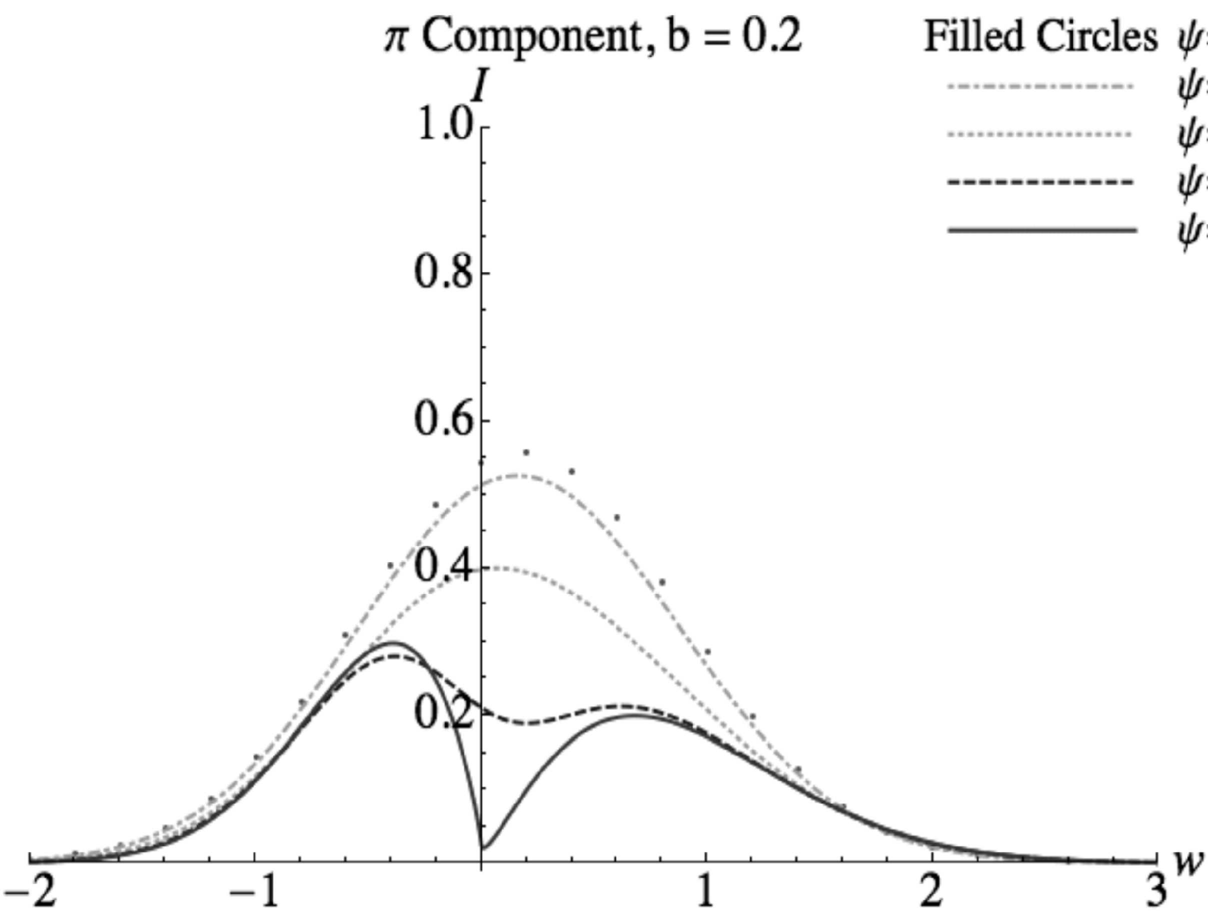

Figure 2: Lorentz-Doppler profiles of π-components of highly-excited hydrogen/deuterium spectral lines calculated by Eqs. (8), (9), for the scaled magnetic field *b* = 0.2 (defined in Eq. (1)) at five different values of the angle of observation y with respect to the magnetic field.

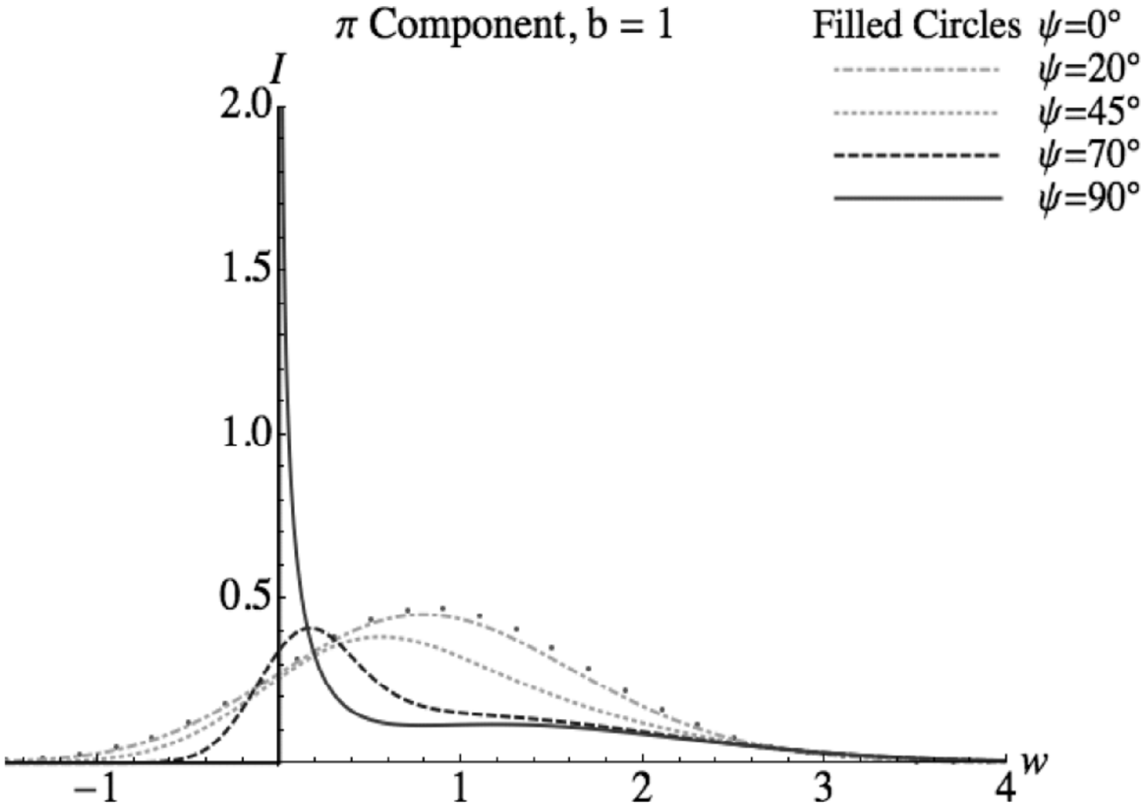

Figure 3: Same as in Figure 2, but for *b* = 1.

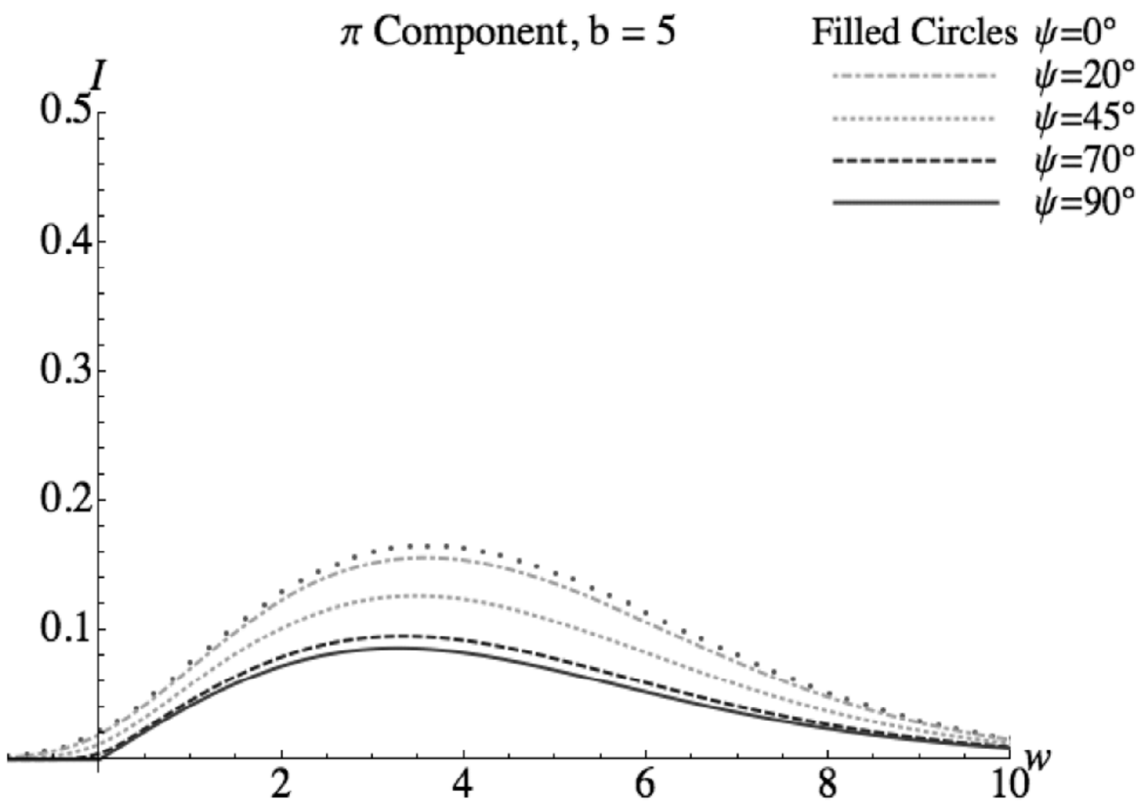

Figure 4: Same as in Figure 2, but for *b* = 5.

Figures 5-7 present the analogous set of Lorentz-Doppler profiles, but for σ-components of HHSL.

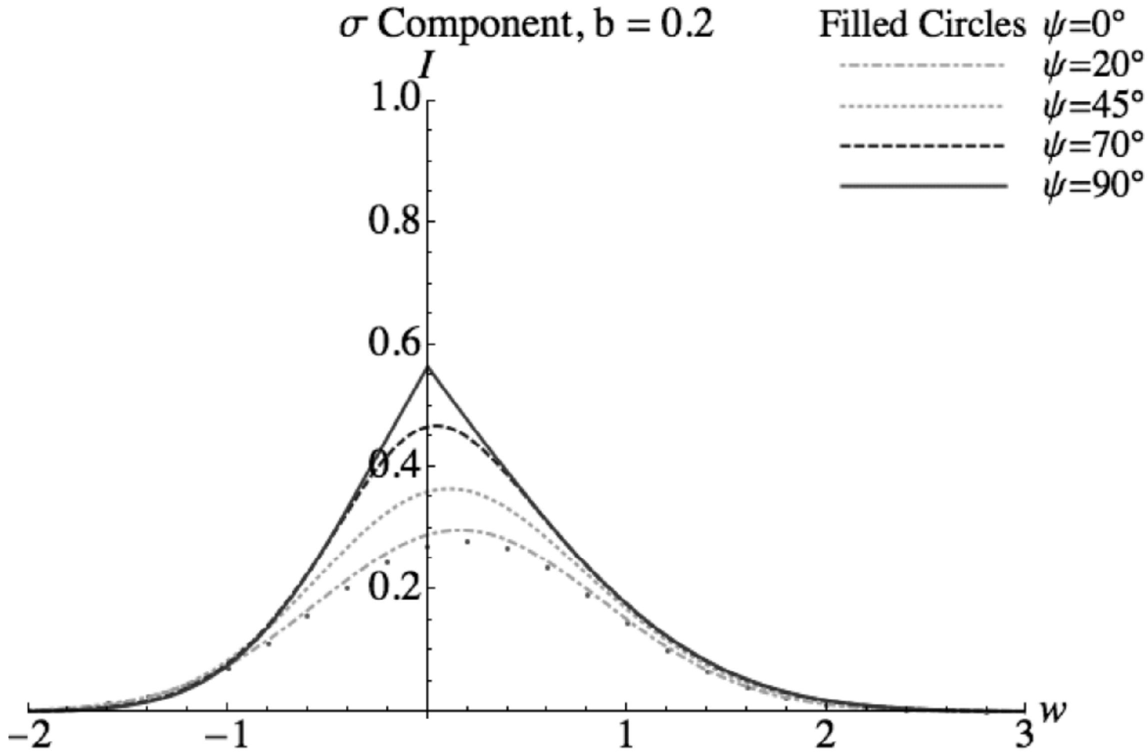

Figure 5: Lorentz-Doppler profiles of σ-components of highly-excited hydrogen/deuterium spectral lines calculated by Eqs. (8), (9), for the scaled magnetic field *b* = 0.2 (defined in Eq. (1)) at five different values of the angle of observation y with respect to the magnetic field.

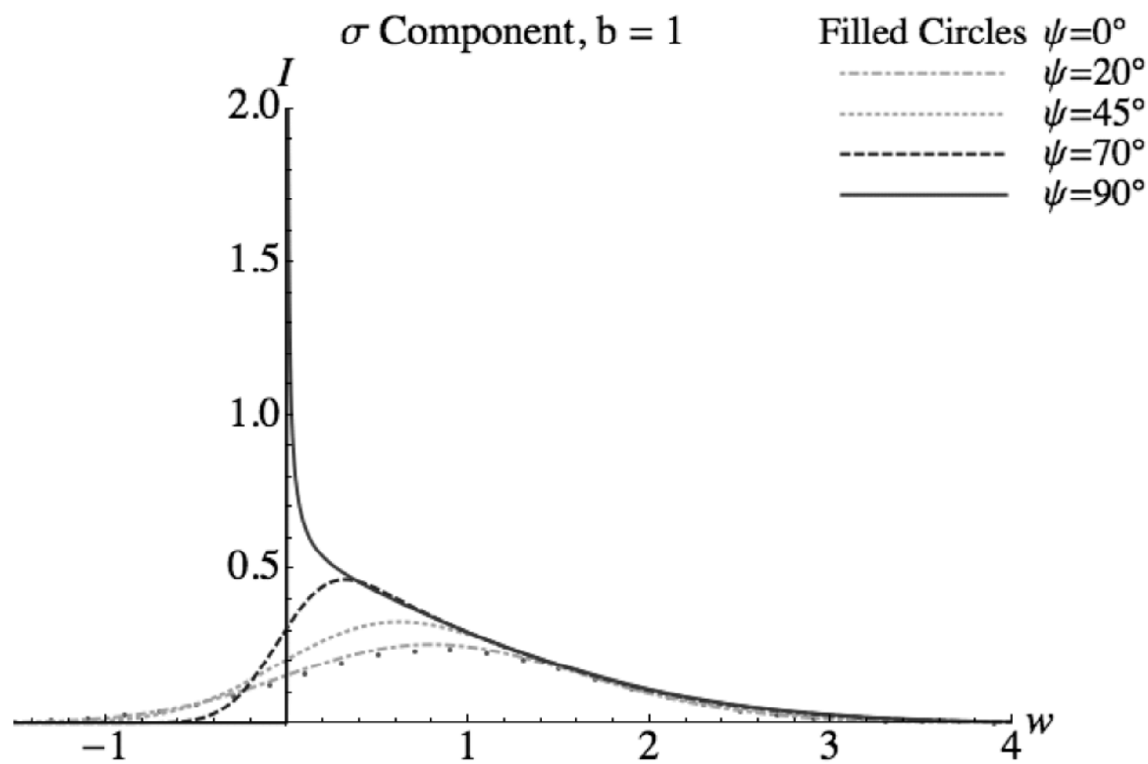

Figure 6: Same as in Figure 5, but for *b* = 1.

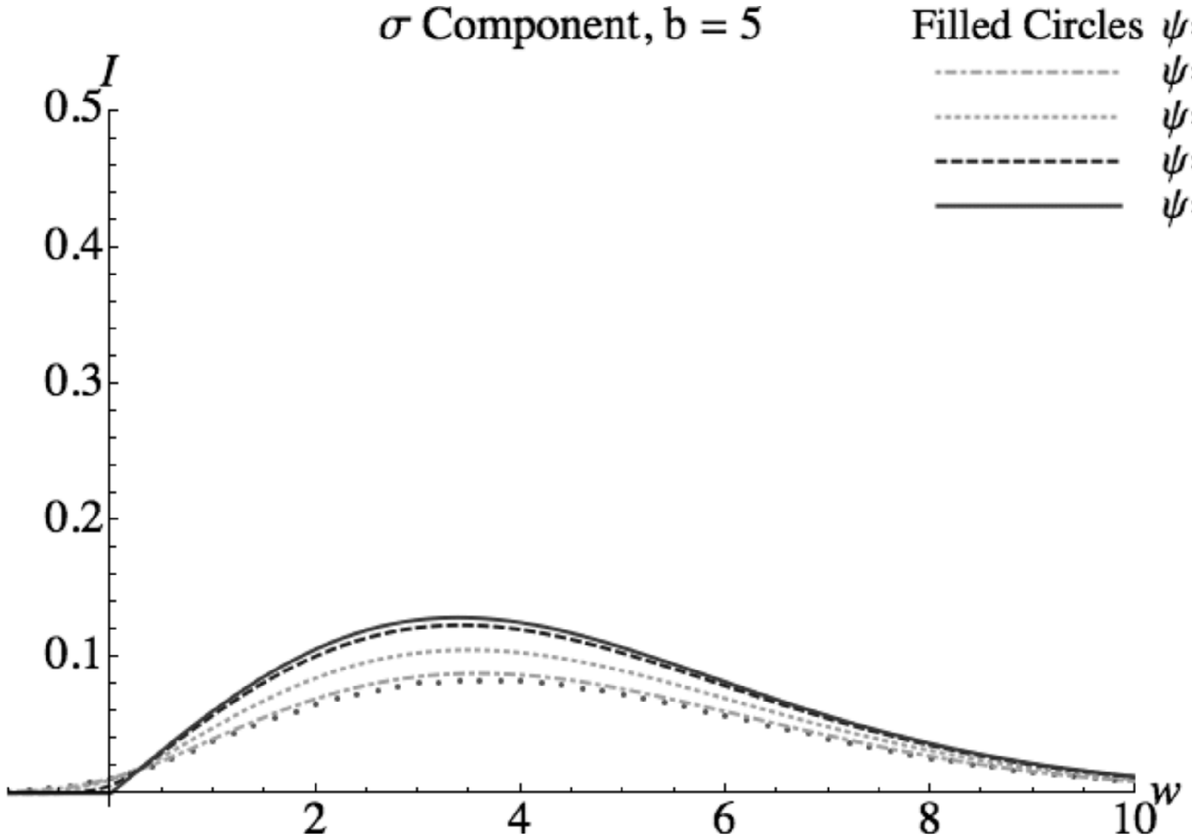

Figure 7: Same as in Figure 5, but for *b* = 5.

One of the purposes of the present study was to see the evolution of the effect of the suppression of the π-components (compared to the σ-components) as the angle of the observation ψ decreases from 90 degrees, *i.e.*, as ψ decreases from the value, at which this effect was discovered in paper [1]. By comparing Figure 4 and Figure 7 (both of which correspond to $b = 5$, *i.e.*, to the relatively strong magnetic field) we can deduce the following. As ψ decrease from 90 to 70 degrees, the effect of suppression, while diminishing, is still present. However, already at ψ = 45 degrees, the effect of the suppression is absent. Thus, we arrive to the conclusion that the suppression of the π-components (compared to the σ-components) occurs at the perpendicular or close to the perpendicular direction of observation (with respect to the magnetic field *b*), but disappears at lower values of the angle of the observation.

3. CONCLUSIONS

We extended the analytical results for the Lorentz-Doppler profiles of HHSL for the arbitrary strength of the magnetic field *b*, obtained in paper [1] for observations parallel or perpendicular with respect to *b*, to arbitrary angles of the observation ψ. For this more complicated general case, we managed to reduce the analytical result to just a single integral.

We studied the evolution of the effect of the suppression of the π-components (compared to the σ-components) as the angle of the observation ψ decreases from 90 degrees, *i.e.*, as ψ decreases from the value, at which this effect was discovered in paper [1]. We found that the suppression of the π- components (compared to the σ-components) occurs at the perpendicular or close to the perpendicular direction of observation (with respect to the magnetic field *b*), but disappears at lower values of the angle of the observation.

The results of the present paper should be important, *e.g.*, for spectroscopic diagnostics of edge plasmas of various magnetic fusion devices around the world.

APPENDIX

Corrections to Figures 9 and 10 from paper [1]

In Figures 9 and 10 from paper [1], the profiles of π-components were not to scale. The corrected Figures 9 and 10 are presented below as Figures. A.1 and A.2, respectively.

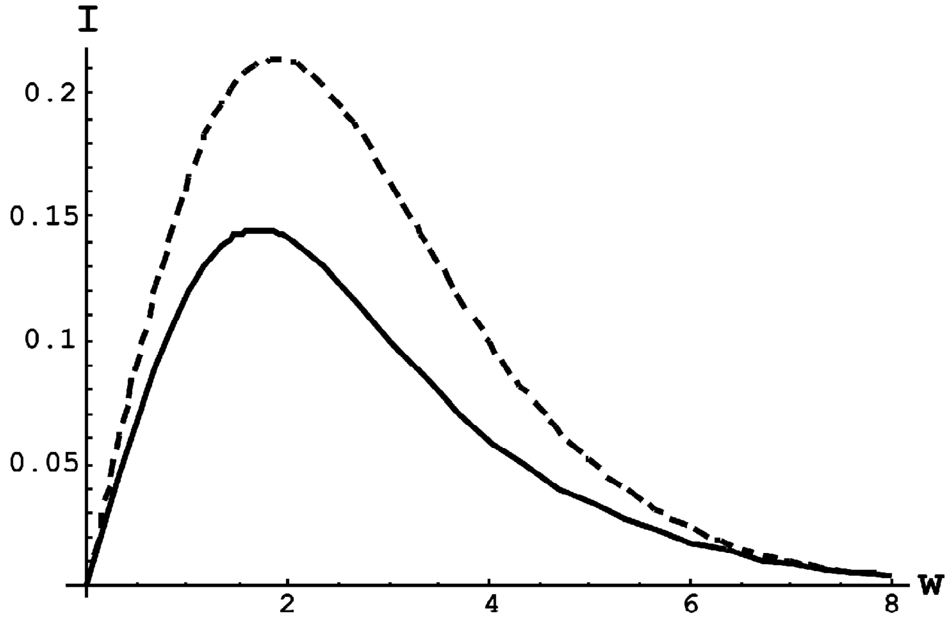

Figure A.1. (Corrected Figure 9 from [1].) The Lorentz-Doppler profile of a *p*-component (solid line) and of a *s*-component (dashed line) of a hydrogen line calculated according to Eqs. (26), (29) from [1] for the scaled magnetic field *b* = 3. The direction of observation is perpendicular to the magnetic field *b*.

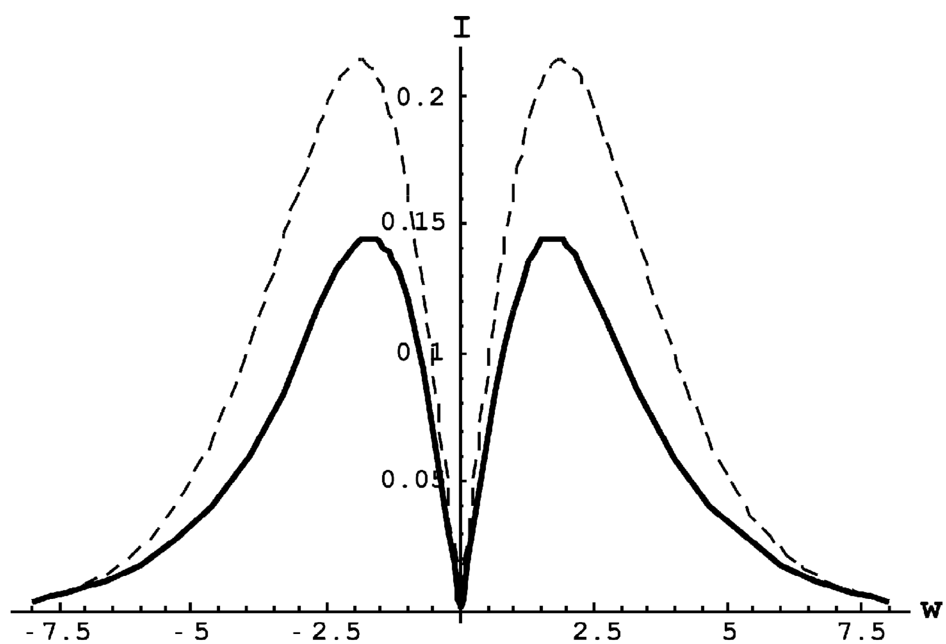

Figure A.2. (Corrected Figure 10 from [1].) The resulting Lorentz-Doppler profile for a pair of Stark components, corresponding to two equal by magnitude and opposite by sign values of X_{ab} (in Eq. (8) from [1]). Specifically, it is the combined profile corresponding to the cases of *b* = 3 and *b* = –3 for the observation perpendicular to the magnetic field *b*. Solid line: the pair of *p*-components. Dashed line: the pair of *s*-components.

It should be also noted that the halfwidth of the Lorentz-Doppler profiles is a non-monotonic function of the scaled magnetic field *b for observations perpendicular to b*. As $|b|$ increases from zero, the halfwidth first decreases, then reaches a minimum at $|b| = 1$ (*i.e.*, when the shift in the Lorentz field is equal to the Doppler shift), and then increases. *This is a counterintuitive result.*

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